The Holographic Principle and the Nature of Reality

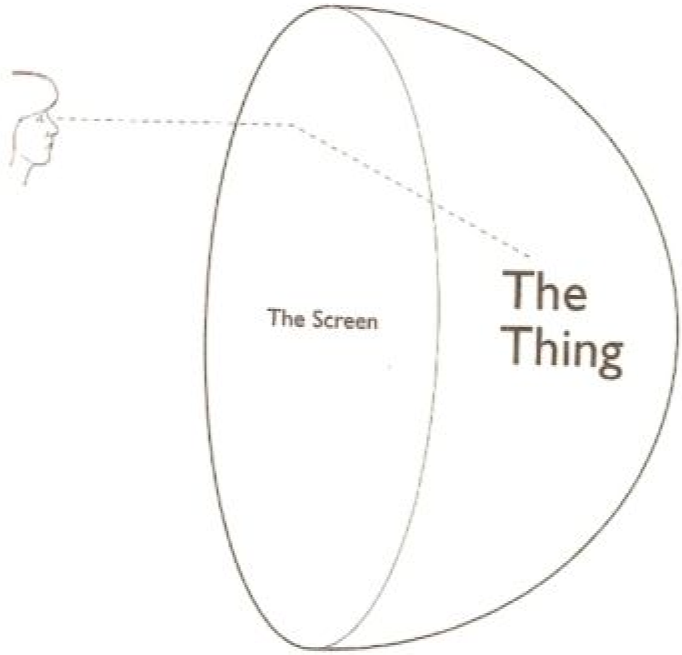
Introduction

This article discusses the nature of reality in the context of the holographic principle of quantum gravity. The goal of this article is to explain the holographic nature of the physical world. Along the way, modern physics is discussed at great length. Although it will appear that this is a discussion about the foundations of physics or fundamental physics, nothing could be further from the truth. There really is no such thing as fundamental physics. For example, quantum gravity is often referred to as a theory of everything, but in reality there is no such thing as a theory of everything. Instead, what modern physics has in its bag of tricks are principles that are sort of fundamental, but even the idea of a fundamental principle is wrong.

The Holographic Nature of the Physical World

The holographic principle is the most fundamental thing we know about the nature of the world. When I refer to the world, I mean the observable physical universe, which is really a holographic world. The most fundamental thing we know about the observable physical universe is that it is a holographic world that is characterized by the holographic principle. This isn't just me talking, but this is the overwhelming consensus of theoretical physicists that work in the area of quantum gravity. If you don't believe me, read Amanda Gefter's recent book Trespassing on Einstein's Lawn where she interviews many of the most prominent theoretical physicists in the world that work in the area of quantum gravity. They all pretty much say the same thing. You could also watch the videos that are listed at the end of this article, where again many prominent theoretical physicists say exactly the same thing. That doesn't make the holographic principle a theory of the world. It's a principle, like the principle of relativity or the equivalence principle or the uncertainty principle. In some sense, the holographic principle is the most fundamental of all the physical principles and supersedes all other principles. That does not make it a theory of everything. There is no such thing as a theory of everything.

The reality we perceive in some sense is emergent from a more fundamental description of reality, much like the appearance of a virtual reality is emergent from a computer screen. The holographic principle is all about describing the nature of the computer screen. What we really want to know is what underlies the holographic principle. What is more fundamental than the computer screen? The answer that we'll inevitably be driven to is that the consciousness of the observer is more fundamental than the information encoded on the computer screen. If we really want to talk about the fundamental nature of reality, we have to talk about consciousness.



The Observer, the Screen and the Thing

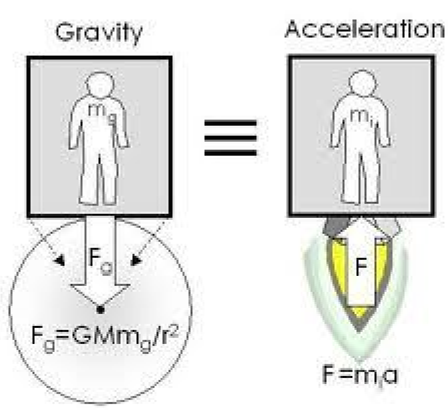
If we really want to understand consciousness, we need to listen to the testimony of enlightened beings like Nisargadatta Maharaj. Enlightened beings tell us there are only two things about the nature of observable reality that we can know with certainty. The first thing is that a fundamental description of observable reality must be observer-centric, in the sense that the observer is at the central point of view of whatever it observes in its own observable world. The second thing is the nature of observation can best be described by the concept of holographic projection, in the sense that all observable forms of information are projected like images from a screen to the observer's central point of view. The irony is that an enlightened being would never refer to observable reality as *reality*, but rather as an illusion, like a dream or a virtual reality. Only the observer itself has an underlying or ultimate reality, which we call consciousness.

Everything written in this article is an attempt to bring some kind of scientific understanding to what enlightened beings are telling us about reality. This kind of scientific understanding by its nature must be conceptual since it's based on scientific concepts. The ultimate understanding one gains when one becomes enlightened is not conceptual. One must realize that for oneself.

The Mathematical Structure of Space-Time, Particle Physics and Quantum Theory

The idea of relativity theory is based on the principle of relativity and the equivalence principle. The principle of relativity tells us that there is a maximal rate of information transfer in three dimensional space, which we call the speed of light. The speed of light is the same constant for all observers, independent of their state of relative motion. The equivalence principle tells us that the force of gravity is equivalent to an acceleration. When we lump all the so-called fundamental forces together in a theory of quantum gravity, the equivalence principle tells us that all forces are equivalent to accelerations. Einstein used these two principles to formulate the theory of general reality which culminated in his field equations for the space-time metric.

There is a lot going on here that we need to unpack. The first thing is that forces aren't really fundamental. In some sense there really isn't any such thing as a force. Instead, accelerations are fundamental, but what do we really mean by an acceleration. The principle of equivalence is describing an observer in an accelerated frame of reference. We could say that the accelerated reference frame is the observer, at least in the sense of relativity theory. We imagine the observer is at the central point of view or origin of some coordinate system like the x-y plane that is undergoing an acceleration. To make it a three dimensional coordinate system we call it x-y-z, which are the three independent directions of motion of a three dimensional space that we seem to be able to move around within. We can call these three independent directions of motion forward-backward, right-left and up-down. When this coordinate system itself is undergoing accelerated motion we call it an accelerated frame of reference and imagine an observer at the origin or central point of view of the coordinate system that makes observations within this frame of reference. Due to the observer's own acceleration, objects in space that are not accelerated or that accelerate in some different way will appear to have forces that act on them and that cause them to accelerate relative to the observer. The observer in its own accelerated reference frame will observe these objects to accelerate and so will say some force like the force of gravity must act on the object. In reality, there are no forces, only relative accelerations.

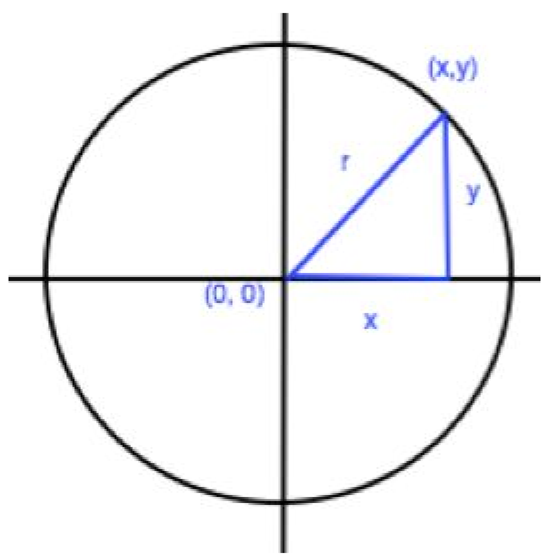


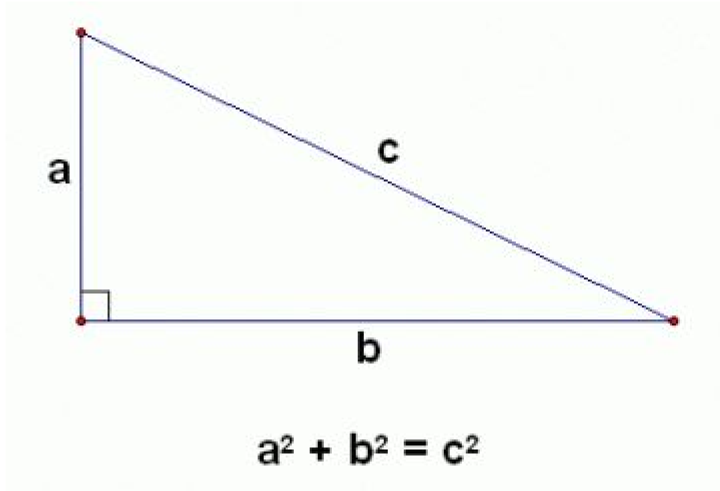
Principle of Equivalence

The classic example is an observer in an accelerating rocket-ship. If that observer drops an object, that object will appear to accelerate towards the floor of the rocket-ship as observed by the accelerating observer, but this is really no different that an observer standing on the surface of the earth that drops an apple and watches the apple accelerate towards the ground. In the second case we say the force of gravity made the apple accelerate. In the first case we don't say any such thing. There are no forces in the first case, only an accelerating rocket-ship. In some sense the force of gravity is an illusion that only arises because the observer is in an accelerated frame of reference. That's exactly what the principle of equivalence is saying.

The second thing we have to unpack is what we really mean by space-time. In relativity theory, space-time is a unified four dimensional geometry represented by an x-y-z-t coordinate system with the observer at the origin. Einstein's great insight was that different coordinate systems can move relative to each other with a velocity, v, and that relative velocity can change and result in accelerations. Einstein postulated that no matter how observers move relative to each other, there is a single invariant quantity that all observers will agree upon when they make a measurement. That invariant quantity is called the proper-time, τ, and is mathematically formulated as $(\Delta\tau)^2=(\Delta t)^2-[(\Delta x)^2+(\Delta y)^2+(\Delta z)^2]/c^2$. This formula looks exactly like the Pythagorean theorem that measures the straight-line distance between two points in the x-y-z-t coordinate system, except it has a funny minus sign that distinguishes spatial coordinates from time. This minus sign turns out to have profound implications about the nature of the world.

It's worth making a brief comment about the Pythagorean theorem as it plays a critical role in all of modern physics. If we have an x-y coordinate system and measure some distance x along the x-axis and some distance y along the y-axis, the total length D of the straight-line distance from the origin (0,0) of that coordinate system to the point located in the x-y plane that is specified by the coordinates (x,y) is given by $D^2=x^2+y^2$. This is the length of the hypotenuse of a right-angled triangle, which is the side opposite to the right angle. By definition, the right angle indicates that the y-axis is orthogonal to the x-axis with a 90 degree right angle between them. Since words can be confusing, it's best to look at a geometric representation of this statement.





Pythagorean Theorem

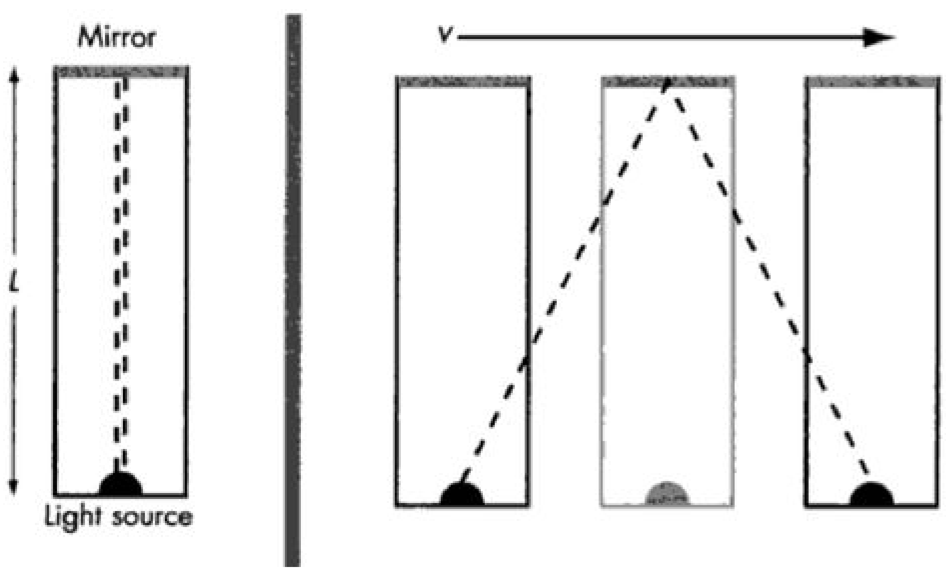
Einstein postulated that this straight-line distance between two points is the only invariant that all observers will agree upon when they make a measurement. In a flat space-time geometry, the straight-line distance between two points is the shortest distance between those two points. In Einstein's view of things, this is the only distance measurement that all observers can agree upon. When space-time geometry becomes curved, as it does when the effects of forces like gravity are at work, which is to say different observers move relative to each other with accelerated motion, then the concept of proper-time has to be generalized as the shortest possible distance between two points in that curved space-time geometry. A point in the coordinate system is labeled with coordinates (x,y,z,t). Einstein thought of these points as actual events that could be measured, like the position in space (x,y,z) of a point particle that is measured at some time t. Einstein was interested in the relation between different events in space-time, such as two events labeled by different points 1 and 2. He postulated that the only invariant measurement that all observers would agree to be the same was the proper-time interval between these two events.

The first thing to point out is that in its own coordinate system, the observer is at rest relative to itself, so the observer's proper-time is ordinary time $\tau=t$. As observed from another coordinate system that moves with velocity, v, relative to the first observer's coordinate system, the proper-time interval is given by the more general formula. We can immediately use this relation to derive the formula for time dilation. Let's take the two events to be two clicks of a clock the first observer carries with itself. That observer observes the two clicks to be separated by a time interval Δt. As far as the observer at rest relative to itself is concerned, proper-time is ordinary time $\Delta\tau=\Delta t$. From the perspective of the other observer, with a coordinate system labeled as (x',y',z',t'), the clock appears to move a distance $D^2=(\Delta x')^2+(\Delta y')^2+(\Delta z')^2$ between two clicks of the clock. Since that distance is the relative velocity multiplied by the time interval, $D=v\Delta t'$, the second observer observes a proper-time interval $(\Delta\tau)^2=(\Delta t')^2-D^2/c^2=(\Delta t')^2-v^2(\Delta t')^2/c^2$. Since

$\Delta\tau=\Delta t$ is an invariant quantity that both observers measure to be the same value, we have $(\Delta t)^2=(\Delta t')^2[1-v^2/c^2]$. From the perspective of the second observer, the clock carried by the first observer appears to run slower compared to a similar clock carried by the second observer. This result gives Einstein's famous expression for time dilation $(\Delta t')^2=(\Delta t)^2/(1-v^2/c^2)$.

Time dilation is the effect of time appearing to run slower when two observers move relative to each other with constant velocity. Time dilation is implied by the constancy of the speed of light for all observers, which is verified by experimental observation. If two observers carry identical clocks, from the perspective of a stationary observer at rest relative to itself, a clock carried by a moving observer appears to run slower compared to the stationary observer's own clock.

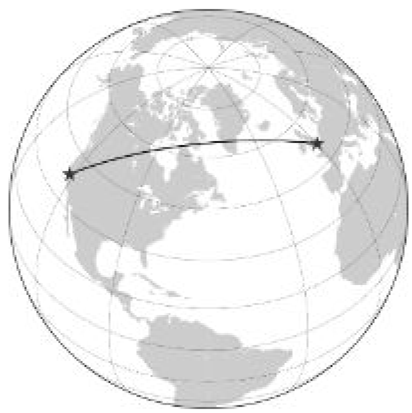
The essential problem is to define how time is measured with a clock. The natural definition of a clock is a light clock, which is a beam of light that bounces back and forth between two mirrors. The time interval it takes the beam of light to bounce back and forth between the two mirrors is a tick of the clock, which is called Δt. If the mirrors are separated by a distance L and the speed of light is c, then the stationary observer's own clock has a time interval $\Delta t=2L/c$.



←------------ $W=v\Delta t'$ ------------→

Now consider a second observer with a light clock that moves relative to the first observer with a velocity v. In a time interval $\Delta t'$, the second light clock moves a distance $W=v\Delta t'$ relative to the first observer. Since the L and W directions are orthogonal, we can use the Pythagorean theorem to calculate the total distance D the light beam travels from the perspective of the first observer as it bounces back and forth between the two mirrors: $(D/2)^2=L^2+(W/2)^2$. Since the speed of light is constant for all observers, $\Delta t'=D/c$, which is the time interval the first observer measures the second observer's clock to tick with as the light beam bounces back and forth between the two mirrors. Putting all the factors together gives $(c\Delta t')^2=(c\Delta t)^2+(v\Delta t')^2$. Recall that Δt is a tick of the first observer's own clock while Δt' is a tick of the second observer's clock as observed by the first observer. This gives the equation for time dilation $(\Delta t')^2=(\Delta t)^2/(1-v^2/c^2)$. The first observer observes the second observer's clock to run slower even though the two clocks are identical.

Time dilation is a result of special relativity, where observers move relative to each other with constant velocity. This result can be generalized to include accelerations, which we call general relativity. With this generalization, the proper-time interval formula is generalized to include factors that are called the space-time metric, which are a measure of the curvature of space-time geometry. The proper-time interval is still the only invariant quantity all observers will agree upon when they make a measurement, but instead of being the straight-line distance between two points or two events in the space-time geometry, Einstein tells us the proper-time is the shortest distance between two points in the curved space-time geometry. For example, the shortest distance between two points on the surface of a sphere is a geodesic or great circle if we are constrained to travel only on that curved surface.



The above statement about the proper-time interval as the shortest distance between two points is not quite correct. The proper-time is like the shortest distance between two points, and would be the shortest distance if not for that funny minus sign, but it's actually a maximal value. The effect of relative motion is to make time appear to run more slowly, hence the effect of time dilation. The correct statement of relativity theory is that we are to maximize the proper-time interval, not minimize a distance between two points, but this is really a distinction without a difference.

Einstein tells us that the only thing we can know about the relation between two events that all observers will agree upon when they measure those events is the proper-time interval. He tells us we have to maximize the proper-time, which is like the shortest distance between two points in the curved space-time geometry. This shortest distance between two points is referred to as a principle of least action, and the proper-time gives rise to the action. When we measure the path of a particle through space-time, we are actually measuring that path point by point, and each segment along the path is characterized by a proper-time interval. Each segment of the path is determined by maximizing the proper-time or minimizing the action. Each point on the path is the measurement of the particle's position in space at some moment of time. Since the particle is at rest relative to itself, its proper-time is ordinary time in its own rest frame. To say the particle follows a path that maximizes proper-time is sort of like saying the particle follows a path that maximizes the observer's perception of ordinary time in that particular frame of reference.

Einstein's formulation of space-time geometry characterized by a proper-time interval that is the only invariant measurable quantity all observers can agree upon when they make a measurement implicitly carries with it the idea of particle physics. Each point in space-time is a possible event, like the measurement of the position of a point particle at some point in space at some moment of time. If the observer measures a sequence of such events, the observer measures the path the particle follows through space-time. Einstein tells us this path is determined by maximizing the proper-time interval. All observers will observe the same path. To say the path the particle follows as it moves through space-time is an invariant that all observers will agree upon as they measure the path of the particle, no matter how those observers move relative to each other as they measure that path, is a statement of causality. The perception of space and time is relative to the motions of observers, such as the effect of time dilation, but the actual observed paths that particles follow as they move through space-time are the same for all observers.

Since the paths particles follow as they move through space-time are determined by maximizing proper-time, which is an invariant quantity all observers agree upon, different observers agree upon the paths particles follow. Different observers measure the same paths. The paths particles follow as they travel through space-time are a sequence of events determined by maximizing proper-time. Each event is the measurement of the particle's position in space at some moment of time. To calculate the proper-time in the general case of accelerations, we need to know the curvature of the space-time geometry, which is given by the space-time metric. Einstein also gives us the equations for the space-time metric, which are called Einstein's field equations.

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

Einstein's Field Equations for the Space-Time Metric

There is a natural feedback in Einstein's formulation of general relativity since the paths particles follow through space-time are determined by maximizing proper-time and proper-time depends on the space-time metric, which is determined by solving Einstein's field equations. Particles carry mass and energy as they move through space-time, and all mass and energy is gravitating in the sense of curving space-time geometry. This feedback occurs due to the curvature of space-time geometry that results as particles move through space-time. That curvature is determined by solving Einstein's field equations for the space-time metric when gravitating particles move through space-time. The space-time metric is used to calculate the proper-time, which determines the particle's paths through space-time when maximized. The bottom line is that gravity is the curvature of space-time geometry that arises from the motion of particles, and particles follow paths that maximize proper-time as they move through space-time.

Einstein's formulation of relativity in terms of invariant proper-time is what makes four dimensional space-time geometry a unified geometry. The proper-time interval when maximized is like the shortest possible distance between two points in the geometry. To say this proper-time interval is the only invariant quantity of the geometry is like enforcing the Pythagorean theorem on the geometry in the sense of specifying a shortest possible distance between two points, which is the only geometric measurement that different observers can agree upon.

This requirement that proper-time is the only invariant quantity of the geometry is equivalent to a statement that the speed of light is the same for all observers. The speed of light is an invariant quantity because the proper-time interval for a light ray is exactly zero. By definition, the proper-time interval for the straight-line path of a particle is $(\Delta\tau)^2=(\Delta t)^2(1-v^2/c^2)$ where v is the particle's velocity, which is given in the coordinate system as $v^2=[(\Delta x)^2+(\Delta y)^2+(\Delta z)^2]/(\Delta t)^2$. For a light ray, v=c, and so $(\Delta\tau)^2=0$. The proper-time of a light ray is zero in all frames of reference, and so the speed of light is invariant for all observers. Einstein was driven to this concept of proper-time as the only invariant because the experimental evidence is the speed of light is the same value for all observers no matter how they move relative to each other. This is the underlying reason that drove him to the concept of space-time geometry.

Anything that moves at the speed of light has no perception of time. To say the proper-time interval is zero is to say time stands still. Anything that moves at the speed of light also has no rest frame, since light can only travel at the speed of light in all frames of reference. Another implication of the invariance of the speed of light is the speed of light is the maximum speed anything can travel. Understanding the invariant proper-time interval as the shortest possible distance between two points in the space-time geometry tells us speeds greater than the speed of light are impossible, since that distance is zero for a light ray and it makes no physical sense to measure distances less than zero. It's actually worse than a negative distance. The proper-time interval becomes an imaginary number if a particle travels faster than the speed of light.

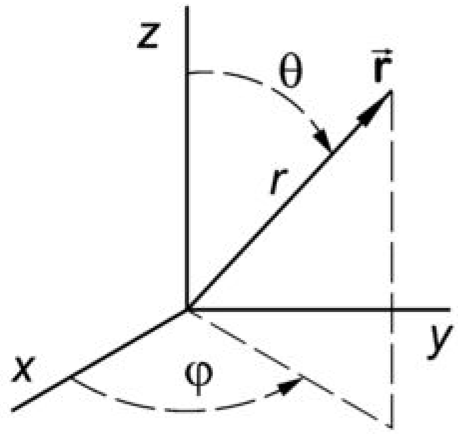
All modern physical theories, like Einstein's field equations for the space-time metric understood as a theory of gravity, or Maxwell's field equations for electromagnetism, are based on the invariance of the speed of light. All modern quantum field theories, like quantum electrodynamics, are based on the invariance of the speed of light. Underlying all these theories is the concept of space-time as a unified four dimensional geometry. This is not just a theoretical nicety. The experimental evidence is the speed of light is the same for all observers.

The special theory of relativity is all about constant relative motion between different observers in different frames of reference. Einstein generalized this theory to include accelerated relative motion between different observers. The basic idea of the principle of equivalence is there is no difference between what we call a force like the force of gravity and the relative accelerations between different observers in different accelerated frames of reference. As far as an observer in an accelerated frame of reference is concerned, forces only appear to act on objects because those objects move with different accelerations relative to the acceleration of the observer.

This idea led Einstein to discover his field equations for the space-time metric. The space-time metric is a measure of the apparent curvature of space-time geometry that arises in an observer's accelerated frame of reference. A different observer in a different accelerated frame of reference will measure a different apparent curvature of space-time geometry and so in effect uses a different metric. The metric is like a system of clocks and rulers that allows the observer to measure the apparent curvature of space-time geometry from the observer's own accelerated point of view. A different observer will measure a different apparent curvature of space-time geometry from that different accelerated point of view. There is however a quantity that all observers will agree upon as having the same value when they measure it. That invariant quantity that is measured to be the same for all observers is the proper-time interval.

This is where Einstein's field equations for the space-time metric feedback into the definition of the proper-time interval, which is defined in terms of the metric. The metric $g=g(x,t)$ is a 4x4 tensor that in any given coordinate system (x,t) has sixteen values g_{ab} with $a,b=0,1,2,3$ that correspond to how distance is measured in a four dimensional curved space-time geometry. The notation is to label time as the 0^{th} coordinate with $x_0=ct$ and to label the x,y,z coordinates as x_a with $a=1,2,3$. For simplicity, (x,t) represents all four coordinates. In terms of these sixteen values for the metric, the proper-time interval is defined as $(\Delta\tau)^2=g_{00}(\Delta t)^2+...$ Measurement of proper-time is independent of the coordinate system we choose to measure things within, but our description of how we make those measurements can change as the coordinate system changes. We can always define a different system of clocks and rulers. When our (x,t) coordinate system description of space-time geometry changes, the sixteen values of the metric as specified in that coordinate system also change, but in such a way as to preserve the value of the proper-time interval since proper-time is the only invariant measure of distance in space-time geometry.

Shortly after Einstein discovered his field equations for the space-time metric, an exact solution was found by Karl Schwarzschild. The Schwarzschild solution describes the gravitational field of an isolated mass M as observed by a distant observer. This observer maintains a constant distance away from the mass, like an observer in a rocket-ship that uses the force of its thrusters to accelerate away from or hover over the mass and avoid being pulled into the mass by the force of gravity. The Schwarzschild metric is defined in this particular (x,y,z,t) coordinate system, which defines the observer's accelerated frame of reference. This coordinate system is rewritten in terms of spherical coordinates as (r,θ,φ,t), where the mass M is located at the origin, and the radius r measures distance from the origin. The solution is written in terms of a specific radius called the Schwarzschild radius, which is given in terms of the mass M as $r=r_s=2GM/c^2$. The Schwarzschild radius has an important role to play in the description of black holes.

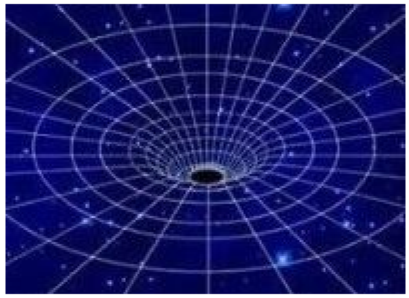


$$ds^2 = -c^2 d\tau^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

$$g_{\mu\nu} = \begin{pmatrix} -\left(1 - \frac{r_s}{r}\right) & 0 & 0 & 0 \\ 0 & \frac{1}{\left(1 - \frac{r_s}{r}\right)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

Schwarzschild Metric

Einstein's field equations for the space-time metric allow us to calculate the metric in some particular coordinate system, but we could also calculate the metric in some other coordinate system. Whatever coordinate system we use to calculate the metric within, when we plug that metric into the equation for the proper-time interval, we have to get the same value for the proper-time independent of whatever coordinate system we used. Although it's often stated that the force of gravity is the curvature of space-time geometry as represented by the metric, that's not exactly right. The apparent curvature of space-time geometry as represented by the metric depends on the coordinate system we use. Use a different coordinate system, and you get a different metric and a different curvature. The more accurate statement is the apparent curvature of space-time geometry and the apparent force of gravity is a consequence of different observers accelerating differently in different accelerated frames of reference.



Curved Space-Time Geometry

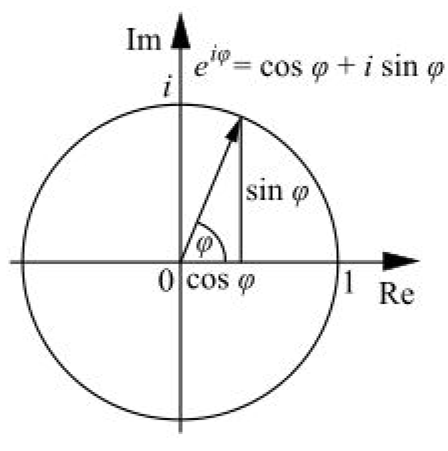
The concept of proper-time not only takes us backward to classical physics, but also takes us forward to quantum physics. The key idea is the concept of action, which is directly proportional to proper-time. In classical physics, action is defined in terms of the kinetic energy and potential energy of a particle as $\Delta A=[KE-PE]\Delta t$, where Δt is a time interval over which the particle moves on some path. For example, a particle of mass m at position x moving with a velocity $v=\Delta x/\Delta t$ has a $KE=\frac{1}{2}mv^2$. If that particle is acted on by the force of gravity F=mg at a distance x above the surface of the earth, where g is the acceleration due to gravity, the particle has a PE=mgx. The principle of least action says the particle will move in such a way as to minimize the action. The particle follows the path that minimizes the action. It turns out the principle of least action is equivalent to Newton's law of motion, F=ma, which in this case says the particle's acceleration in the x-direction toward the ground, $a=\Delta v/\Delta t$, is the acceleration of gravity a=g.

Newton's law of motion is a statement of determinism, as it says when a force acts on a particle, the particle accelerates as a direct effect of that force as F=ma. On the other hand, the principle of least action says the particle could in principle follow any path, but for some unknown reason chooses to follow the path that minimizes the action. Instead of strict determinism, there seems to be the option of choice. This option of choice is what opens up the way to quantum theory.

The Schwarzschild solution of Einstein's field equations for the space-time metric, when a single gravitating mass M is the source of the curvature of space-time geometry, gives the leading terms for the proper-time interval as $(\Delta\tau)^2=(1-2GM/rc^2)(\Delta t)^2-(\Delta r)^2/c^2$, where r is the distance from the center of the mass M to a point in space. By the leading terms, we mean that all particle velocities are much less than the speed of light. If that mass has a radius R and we measure a distance x above the surface of that mass with r=R+x, where R is much greater than x, then the leading terms are $(\Delta\tau)^2=(1+2GMx/R^2c^2)(\Delta t)^2-(\Delta x)^2/c^2$. Defining the acceleration of gravity at the surface of the mass M as $g=GM/R^2$ and locating a particle of mass m at the point x that moves in the x-direction with velocity $v=\Delta x/\Delta t$, then gives $(\Delta\tau)^2=[1+2gx/c^2-v^2/c^2](\Delta t)^2$. Taking the square root, again to leading order, gives $\Delta\tau=[1+gx/c^2-½v^2/c^2]\Delta t$. Defining the action as $\Delta A=-mc^2\Delta\tau=[½mv^2-mc^2-mgx]\Delta t$ and comparing this result to the previous expression for the classical action $\Delta A=[KE-PE]\Delta t$ gives $KE=½mv^2$ and $PE=mc^2+mgx$. Einstein's approach not only recovers classical physics, but also gives us the mass energy.

The classical action for the motion of a point particle in effect is the proper-time interval, which Einstein tells us is the only invariant measurement that all observers will agree upon. The exact relation is given by $\Delta A=-mc^2\Delta\tau$. The classical motion of particles is determined by the path of least action, which is like the shortest distance between two points in a space-time geometry and is the result of maximizing the proper-time interval. This is how classical Newtonian physics is integrated into Einstein's relativistic view of things, but this view of things also opens up the way to quantum theory. A point particle does not necessarily have to follow the path of least action, but also has the potential to follow other possible paths. The motion of the particle has some wiggle-room. Quantum theory introduces the option of choice.

The principle of least action allows us to move forward to quantum theory. The basic idea is to allow the particle to move along any possible path, but with a probability factor that depends on the action. This probability factor is the essence of the wave-function.



Complex Plane and Euler's Formula

Defining the wave-function is really the only difficult thing about quantum theory. Defining the wave-function is the only conceptual barrier in terms of understanding quantum theory. Once we define the wave-function, everything else is pretty much obvious. The wave-function is defined in terms of Euler's formula, which says the exponential function of an imaginary number is the same as a complex trigonometric function. Specifically, $\exp(i\theta)=\cos\theta+i\sin\theta$, where i is the unit imaginary number that results from the square root of -1, or equivalently, $i^2=-1$. Euler's formula has a natural geometric interpretation in the complex plane, which is like the x-y plane where x is the direction of real numbers and y is the direction of imaginary numbers. In the complex plane, we draw a unit-length vector called $z=x+iy=\exp(i\theta)=\cos\theta+i\sin\theta$, where the vector z makes an angle θ with the x-axis, and where $x=\cos\theta$ and $y=\sin\theta$.

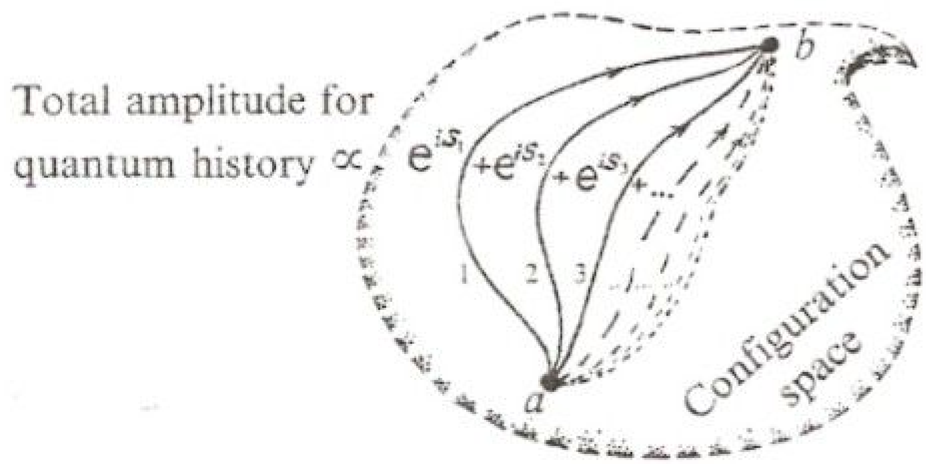
This sounds too simple, but the wave-function is defined as $\Psi=\exp(i\theta)$. This represents a wave because the sine and cosine functions oscillate like waves. The angle θ represents some variable over which the wave-function oscillates. Since we're talking about particle physics and the paths that particles can be observed to follow as they move through space-time, this wave-function represents a probability factor that specifies the probability with which any particular path can be measured. The whole idea of quantum theory is to allow particles to move along any possible path, but with a probability factor that is called the wave-function.

The essence of quantum theory is particles are allowed to move along any possible path, but with a probability factor we call the wave-function. We use the wave-function to determine the probability the particle can be observed to follow a specific path. We first have to calculate what the wave-function is for some specific situation, and that requires we solve some wave equation. This is just like solving Einstein's field equations for a particle we call the graviton, except we are solving some other wave equations for some other assortment of particles. For example, we could solve Maxwell's wave equation for electromagnetic radiation, which is the particle we call the photon, or we could solve Dirac's equation for the electron.

Once we solve the wave equation for the wave-function of some particle, we can use the solution to calculate the probability with which that particle can be observed to follow some path through space-time. There seems to be something circular about this approach, since the wave-function solution for Einstein's field equations both characterizes the nature of space-time geometry in the sense of curvature, but also seems to specify the probability with which the graviton can be observed to follow some path through space-time. We'll have to come back to this problem later. For now let's just focus on the wave-functions and the wave equations for other particles.

How do we discover these wave equations? Richard Feynman gave the answer in terms of the sum over all possible paths formulation of quantum theory. Instead of writing down the wave equation for the particle's wave-function, we can formulate the quantum state of the particle in terms of a sum over all possible paths the particle can follow as it moves through space-time. Each possible path is weighted with a probability factor $\exp(i\theta)$. This is where the rubber meets the road. We have to understand what the angle θ represents. This is where the action principle makes its dramatic entrance onto the stage.

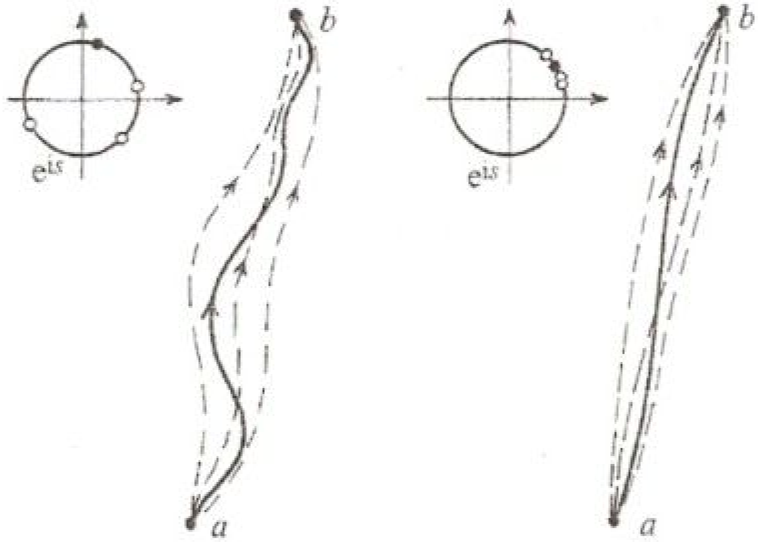
The angle θ is defined in terms of the action as $\Delta\theta=\Delta A/\hbar$ where $\Delta A=[KE-PE]\Delta t$ and $\hbar=h/2\pi$ is the angular version of Planck's constant. This is the action for a particle that moves along the segment of some path over a time interval Δt with a kinetic energy KE and potential energy PE. Feynman's instruction about how to construct the quantum state of the particle is to sum over all possible paths the particle can follow through space-time and multiply each path with the factor exp(iθ), where the angle θ is determined in terms of the action. In the sum over all possible paths formulation of quantum theory, space-time is often referred to as a configuration space within which the particle can move. The quantum state is then given by the sum over all possible paths weighted with the probability factors P=exp(iθ), where P depends on the action for that path.



Sum over all Possible Paths Formulation of Quantum Theory

The classical limit of quantum theory is easily obtained from Feynman's sum over all possible paths formulation. As Einstein told us, all we have to do is look for the path that minimizes the action. The path of least action is the classical path. In the sum over all possible paths, the probability factors exp(iθ) that multiply each path in the sum fluctuate wildly for all paths that are not near the path of least action. The fluctuations get worse in the limit Planck's constant goes to zero since $\Delta\theta=\Delta A/\hbar$. Only as the action approaches a minimum value do the fluctuations calm down. Only the paths that are nearby the path of least action add together and contribute to the sum. All the other paths have contributions that fluctuate wildly and tend to cancel out. The wild fluctuations tend to cancel out because they interfere with each other like interfering waves.

The end result of this wave interference is the classical limit defined by the path of least action. Quantum theory implies the principle of least action as the classical limit, which Feynman's sum over all possible paths formulation makes crystal clear. The magic of quantum theory is in defining the wave-function in terms of Euler's equation Ψ=exp(iθ). The wave interference that is inherent in quantum theory is entirely a result of this way of representing the wave-function.



Principle of Least Action

Although it takes some work to prove, Feynman's sum over all possible paths formulation of quantum theory is equivalent to the wave-function and wave equation formulation of quantum theory. For a free non-relativistic particle that moves in the x-direction with a velocity v much less than the speed of light and not acted upon by any forces, $KE=\frac{1}{2}mv^2$ and PE=constant. The action can be rewritten as $\Delta A=[KE-PE]\Delta t=[mv^2-E]\Delta t=[mv\Delta x-E\Delta t]=[p\Delta x-E\Delta t]$ where $p=mv$ is the particle's momentum, $v=\Delta x/\Delta t$ and $E=KE+PE$ is the particle's total energy. Since the particle moves with constant motion, this gives $\theta=A/\hbar=[px-Et]/\hbar$. The particle's wave-function then takes the form $\Psi(x,t)=\exp(i[px-Et]/\hbar)$. For simplicity, we'll only consider a single spatial direction of motion x, but generalization to three spatial dimensions is not hard.

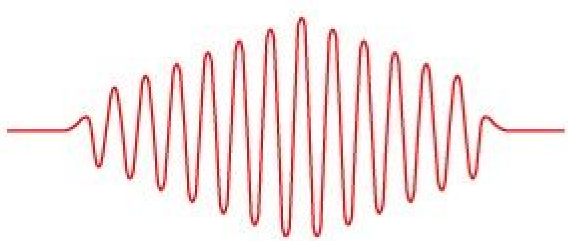
This wave-function satisfies two equations: $i\hbar\Delta\Psi(x,t)/\Delta t=E\Psi(x,t)$ and $-i\hbar\Delta\Psi(x,t)/\Delta x=p\Psi(x,t)$. These two equations are interpreted as operator equations, where the energy operator is defined in terms of a change over time as $E_{op}=i\hbar\Delta/\Delta t$ and the momentum operator is defined in terms of a change over position as $p_{op}=-i\hbar\Delta/\Delta x$. By definition $E_{op}\Psi(x,t)=E\Psi(x,t)$ and $p_{op}\Psi(x,t)=p\Psi(x,t)$. The action of the energy or momentum operator acting on the wave-function, which is called an eigenstate, is to give back the value of energy or momentum, which is called an eigenvalue.

For a non-relativistic freely moving particle, the relationship between energy and momentum is $E=p^2/2m$. The more general relativistic relation is $E^2=p^2c^2+m^2c^4$. In the non-relativistic limit, this expression gives the leading terms $E=mc^2+p^2/2m$. For a massless particle like a photon, this expression simplifies to $E=pc$. Since we can write the particle's energy and momentum in terms of a frequency f and a wavelength λ as $E=hf$ and $p=h/\lambda$, for a massless photon we have $f=c/\lambda$. The factors of Planck's constant cancel out. This is the reason we don't see Planck's constant in the wave equation for a massless particle, like Maxwell's equations for the photon.

In the unified four dimensional geometry of space-time, energy and momentum taken together comprise a four-vector. If we call $E=p_0$, this four-vector p has four components $p=(p_0,p_1,p_2,p_3)$. The relativistic energy-momentum relation is then understood as another expression of the Pythagorean theorem. The length of this four-vector is given as $E^2-(p_1^2+p_2^2+p_3^2)c^2=m^2c^4$. We again have the funny minus sign that distinguishes temporal and spatial dimensions.

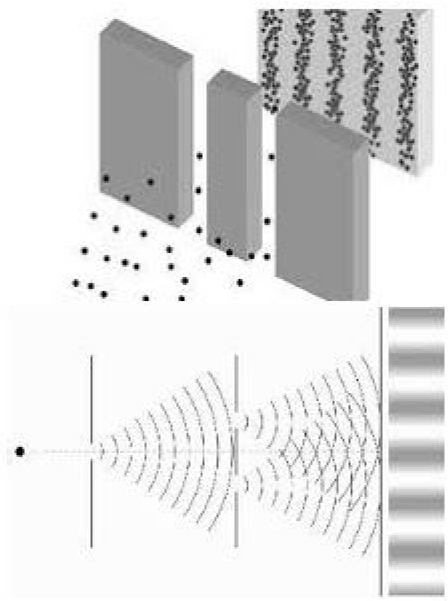
The wave-function $\Psi(x,t)=\exp(i[px-Et]/\hbar)$ can be rewritten in terms of a wave vector k and a wave frequency $\omega=2\pi f$ as $\Psi(x,t)=\exp(i[kx-\omega t])$ where $E=\hbar\omega$ and $p=\hbar k$. The wave vector has a natural interpretation in terms of a wavelength λ as $k=2\pi/\lambda$. There is also a natural definition of wave velocity as $c=\omega/k=\lambda f$. This wave-function obeys a very simple wave equation. This is the simplest possible wave-function for the free motion of a relativistic or non-relativistic particle. For this wave-function, the particle's momentum is exactly determined, but its position in space at which that momentum is measured is totally undetermined. The particle's energy is exactly determined, but the time at which that energy is measured is totally undetermined.

The reason the particle's position in space is totally undetermined when its momentum is exactly measured and its time is totally undetermined when its energy is exactly measured is because this wave-function is totally unlocalized in space and time. If we imagine a wave-function with the form of a wave-packet that to some degree is localized in space and time, then measurement of momentum is uncertain in inverse proportion to uncertainty in measurement of position and measurement of energy is uncertain in inverse proportion to uncertainty in measurement of time. These uncertainties in measurement are expressed by the uncertainty principle as $\Delta p\Delta x\simeq\frac{1}{2}\hbar$ and $\Delta E\Delta t\simeq\frac{1}{2}\hbar$. Uncertainty in measurement is inherent in the mathematical form the wave-function takes $\Psi(x,t)=\exp(i[px-Et]/\hbar)$, where the wave-function is understood as a probability amplitude that specifies the probability with which the particle can be measured at some position in space at some moment of time. A wave-packet can always be constructed by adding wave-functions with different values for energy and momentum. The construction of a localized wave-form by adding wave-functions together with different values of energy and momentum is called a Fourier transformation. In quantum theory, this sum is called a linear superposition of eigenstates.



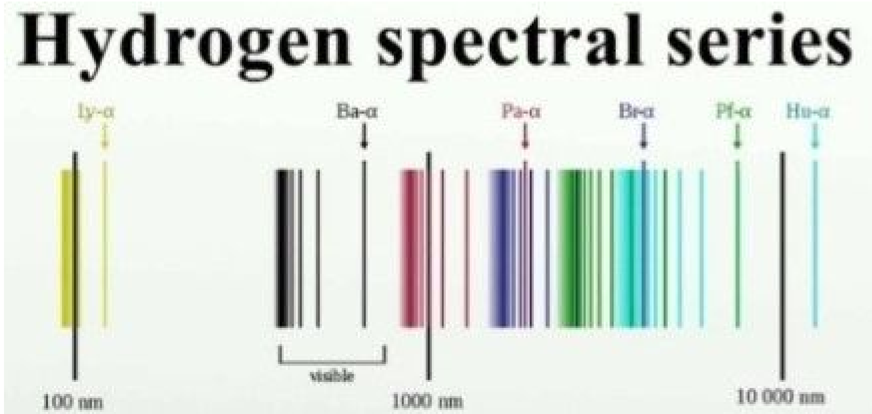
Wave-packet

The upshot is a quantum particle is not a classical particle. A quantum particle is also not a classical wave. Instead, there's a wave-particle duality that's inherent within quantum theory. When the particle's wave-function is well localized, the particle can be accurately measured to be localized at some point in space at some moment in time, and the quantum particle behaves like a classical particle. When the particle's wave-function is not well localized, multiple measurements of the quantum particle look like classical wave behavior when averaged together. This classical wave-like behavior includes phenomena like interference patterns. This is essentially what the double slit experiment is telling us.



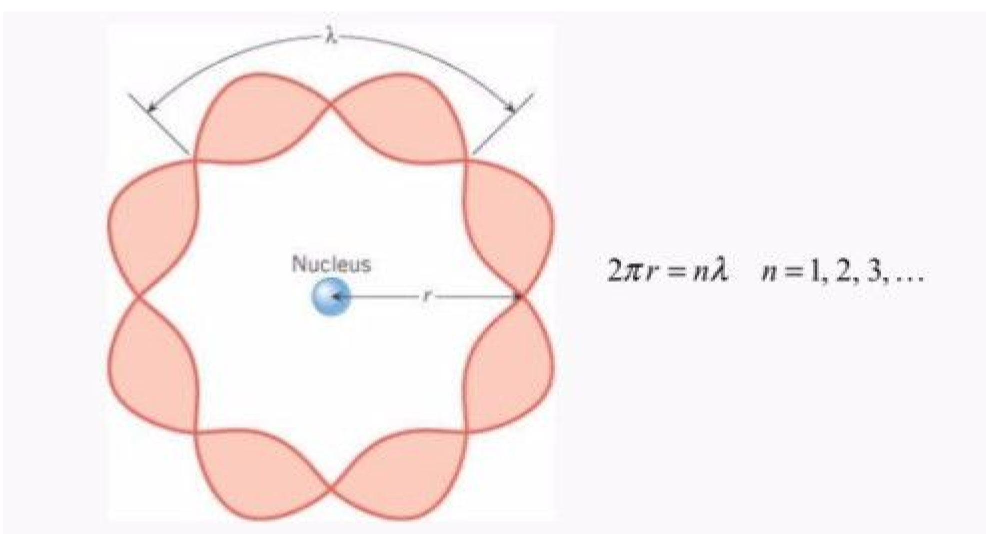
Double Slit Experiment Interference Pattern

The most abstract mathematical formulation of quantum theory is in terms of operators. These operators could be differential operators or matrix operators or even more abstract mathematical objects. A Hamiltonian operator is defined that represents the total energy of some system of interest, such as an interacting collection of particles. For example, we could be interested in the energy levels of the hydrogen atom, which is a bound state of an electron orbiting a proton. The negatively charged electron is bound to the positively charged proton because of electromagnetic interactions, which cause opposite charges to attract through Coulomb's law, which looks very similar to Newton's law of gravity. As is well known, the electron cannot orbit the proton in any possible orbit, but is restricted to discrete quantized orbitals, each of which is characterized by a discrete quantized energy level. We measure these quantized energy levels when we measure the spectrum of light emitted from a gas of hydrogen atoms, which consists of discrete spectral lines.



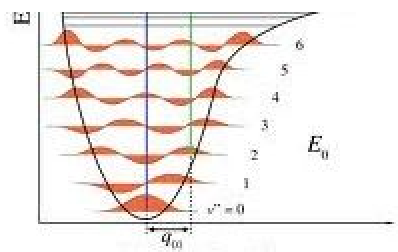
It turns out to be quite simple to calculate the quantized energy levels of the hydrogen atom. As the electron orbits the proton, in the non-relativistic limit the electron's total energy is given in terms of its momentum p and the radial distance r from the proton as $E=p^2/2m-e^2/r$. The value e is the electric charge of the electron. The electron's electrostatic potential energy $PE=-e^2/r$ arises from Coulomb's law just as gravitational potential energy arises from Newton's law of gravity. The minus sign indicates there is an attractive force between the negatively charged electron and the positively charged proton, which allows bound state orbits to form.

Quantum theory tells us we should think of the electron's momentum as a wavelength λ where $p=h/\lambda$. The basic idea of quantum theory is that for a quantized bound state orbit, the wavelength cannot take on any possible value, but must be quantized into discrete values. The circumference of the electron's orbit at a radius r is $2\pi r$. If we fit a discrete number n of wavelengths into this circumference, then $n\lambda=2\pi r$. This gives momentum as $p=nh/2\pi r$. The quantity $L=pr=mvr=nh/2\pi$ is called orbital angular momentum, and is quantized in integer units n=1,2,3,... of $\hbar=h/2\pi$. These quantized values of orbital angular momentum define the Bohr orbitals of the hydrogen atom.



We plug these allowed values for momentum into E and reexpress the electron's total energy in terms of this integer n as $E=(nh/2\pi r)^2/2m-e^2/r$. This gives the total energy simply as a function of the radius r. The basic idea of energy in physics is that energy likes to be minimized. Minimizing a particle's energy is like a particle following the path of least action. We minimize the electron's energy by examining how that energy changes as the radius of its orbit changes.

The equation we need to look at is $\Delta E=[-2(nh/2\pi)^2/2mr^3+e^2/r^2]\Delta r$. For any change in r or value of Δr, we set $\Delta E=0$, which corresponds to the minimum value. This minimum value occurs where the slope of the curve ΔE as a function of r is zero. The minimum value occurs when the slope of the curve $\Delta E/\Delta r=0$, which is like the bottom of a valley. The solution to $\Delta E=0$ gives the allowed quantized values of the electron's orbital radius as $r=(nh/2\pi)^2/me^2$.

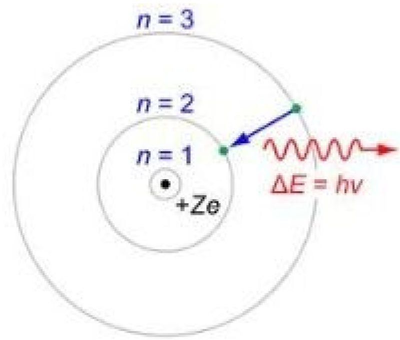


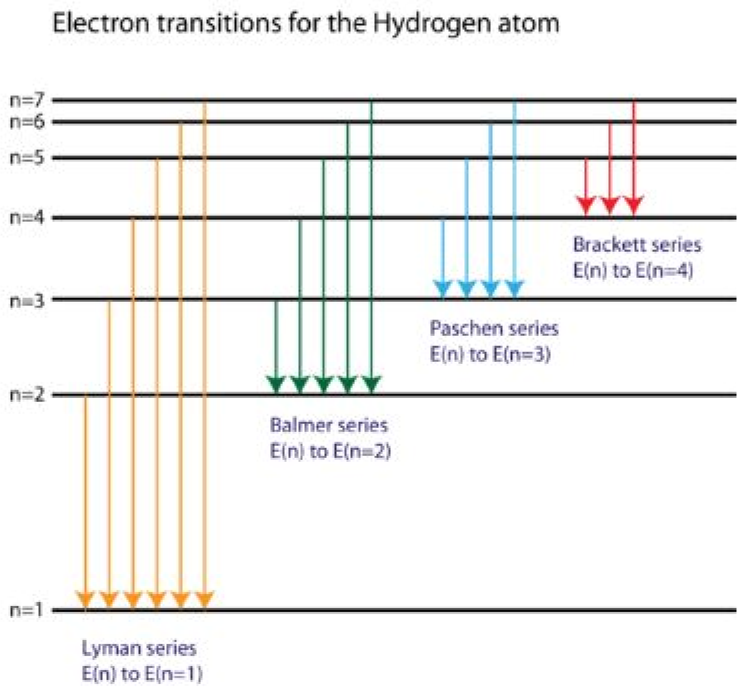
Effective Energy of Hydrogen Atom and Energy Levels

These allowed values of the electron's quantized orbital radius depend on the integer n, which can run as n=1,2,3,... This integer tells us how many wavelengths of the electron's wave-function can fit into the orbital circumference. The smallest orbital radius is called the Bohr radius and has a value of about 0.5×10^{-8} centimeters. This smallest possible orbital radius with n=1 is called the ground state of the hydrogen atom. Values of n greater than 1 are called excited states.

If we plug these allowed values of the orbital radius into the expression for the electron's energy, we find the allowed quantized energy levels of the hydrogen atom $E_n=-2\pi^2me^4/(nh)^2$. In terms of the allowed values of the orbital radius, this is simply $E_n=-e^2/2r$. The minus sign indicates the allowed quantized values of energy correspond to bound states with negative energy. If the electron had a positive energy, it would be able to move around freely, since the electron's escape velocity is defined by E=0. A negative energy indicates the electron is bound to the proton.

These quantized energy levels perfectly match the observed spectral lines of the hydrogen atom. Each spectral line corresponds to a photon that is emitted from the hydrogen atom as the electron makes a transition from a higher energy level characterized by some integer m to a lower energy level characterized by another integer n. The quantized energy of the photon is given in terms of the quantized energy levels of the hydrogen atom as $E=hf=E_m-E_n$. Since m is greater than n, this is a positive energy. The reverse situation happens when the hydrogen atom absorbs a photon. The ionization energy of the hydrogen atom is defined by a transition from the ground state to the free state with E=0, which is simply given by $E=-E_1=2\pi^2me^4/h^2$, which is experimentally confirmed to be about 13.6 electron volts.





This is basically all that non-relativistic physics can tell us about the hydrogen atom. The exact non-relativistic solution is found by solving the Schrodinger wave equation, but in terms of what we can measure experimentally, this heuristic discussion is pretty much all that we can say. If we want to say more, we have to look for a relativistic wave equation.

Paul Dirac looked for such a relativistic wave equation for the electron and found it. Dirac looked at the relativistic relation between a particle's energy and momentum $E^2=p^2c^2+m^2c^4$. For various reasons, he became motivated to take the square root of this equation. In the Schrodinger wave equation, the momentum variable is understood as a quantum operator. Dirac discovered he could take the square root of the energy-momentum relation and understand momentum as a quantum operator, but only if he introduced some new quantum operators.

These new quantum operators are called spin operators. In the simplest case, they're called Pauli spin matrices. A matrix is an array of numbers that obeys some multiplication rules, but a matrix understood as a mathematical object is a non-commuting variable. The order with which the matrices are multiplied together matters. The result of multiplying M_1 times M_2 is not the same as multiplying M_2 times M_1 which is to say $M_1M_2 \neq M_2M_1$. Pauli spin matrices are a special kind of 2 by 2 matrix. The Pauli spin matrices are called SU(2) matrices, where U refers to unitary and 2 refers to two independent directions of rotation, which generate the surface of a sphere of unit radius. Dirac found that he had to generalize the Pauli spin matrices to another kind of special 4 by 4 matrix called gamma matrices. The reason for this generalization is the Dirac equation as a relativistic equation allows solutions not only for the particle called the electron, but also for its antiparticle partner called the position. The gamma matrices not only have to describe the spin of the electron, but they also have to describe the spin of the positron.

$$\boldsymbol{\sigma_1} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{bmatrix}$$

$$\boldsymbol{\sigma_2} = \begin{bmatrix} \mathbf{0} & -\boldsymbol{i} \\ \boldsymbol{i} & \mathbf{0} \end{bmatrix}$$

$$\boldsymbol{\sigma_3} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{bmatrix}$$

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix},$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

Pauli Spin Matrices and Gamma Matrices

Dirac discovered the Dirac equation by taking the square root of the energy momentum relation $E^2=p^2c^2+m^2c^4$. He imagined expressing this square root as $\gamma_0 E=[\gamma_1 p_1+\gamma_2 p_2+\gamma_3 p_3]c+mc^2$, where $p=(p_1,p_2,p_3)$ are the three components of the momentum vector in three dimensional space, $p^2=p_1^2+p_2^2+p_3^2$ and the γ are the gamma matrices. When he took this matrix expression for E and required $E^2=p^2c^2+m^2c^4$, he discovered the gamma matrices obeyed anti-commuting relations of the form $\gamma_1\gamma_2+\gamma_2\gamma_1=0$ and unitary relations $\gamma_1^2=\gamma_2^2=-1$ and $\gamma_0^2=+1$. What Dirac accomplished with the gamma matrices was to deconstruct the Pythagorean theorem into a linear relationship.

This deconstruction is related to the nature of space-time geometry, which is described by the Pythagorean proper-time interval $(\Delta\tau)^2=(\Delta t)^2-[(\Delta x)^2+(\Delta y)^2+(\Delta z)^2]/c^2$. When this Pythagorean relation of space-time geometry is deconstructed into a linear relation, the gamma matrices naturally pop out. The gamma matrices enforce the Pythagorean nature of space-time geometry. The gamma matrices even express the funny minus sign that distinguishes the dimension of time from the spatial dimensions. The gamma matrices are expressing space-time symmetry.

$$\psi^{(\alpha)} = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}.$$

Spinor Wave-function

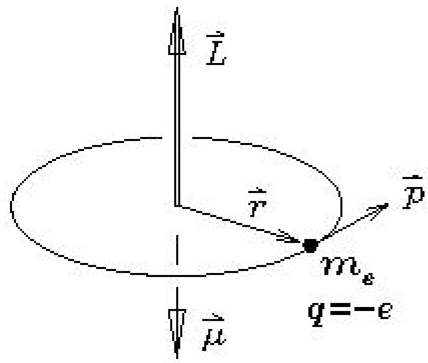
Dirac wrote down a wave-equation for the electron's wave-function Ψ in terms of the gamma matrices as $[\gamma_0 E-(\gamma_1 p_1+\gamma_2 p_2+\gamma_3 p_3)c]\Psi=mc^2\Psi$. This has the form of the Pythagorean theorem, but is linear in E and p. Since the gamma matrices are 4x4 matrices, the spinor wave-function Ψ is a 1x4 column vector. Dirac wrote E and p in terms of differential quantum operators to obtain the wave equation, where E implies a change $i\hbar\Delta/\Delta t$ and p_1 implies a change $-i\hbar\Delta/\Delta x_1$.

$$i\hbar\gamma^\mu\partial_\mu\psi = mc\psi$$

Dirac Equation

What exactly is spin? Angular momentum is a familiar quantity in physics, like the angular momentum of a spinning top or the spinning earth. Spin angular momentum is sort of similar, except it describes the spin of a point particle, which strictly speaking has no dimension or radius around which spin can occur. It turns out that in relativity theory there are some weird properties of space-time when understood as a unified four dimensional geometry that allow point particles to have spin angular momentum. These weird properties have to do with symmetries or with the invariance of the laws of physics under rotation of coordinate systems. These symmetries give rise to conservation laws. The conservation of ordinary angular momentum is due to invariance of the laws of physics under a rotation of a spatial coordinate system, like the x-y plane. Spin angular momentum is also described by a conservation law, which arises from invariance of the laws of physics under a rotation of the x-t plane. Spin is mixing up space with time.

All conservation laws in physics arise from some kind of symmetry. Conservation of energy and momentum arise from symmetry under translation in space and time. Conservation of angular momentum arises from symmetry under rotation in space, while conservation of spin angular momentum arises from symmetry under rotation in space-time. This is an inevitable consequence of making space-time a unified four dimensional geometry.



The upshot is that point particles can carry spin. The electron is a spin ½ particle. The photon is a spin 1 particle. The graviton is a spin 2 particle. All the matter particles carry half integer spin and are called fermions, while the force particles carry integer spin and are called bosons. This is an inevitable result of space-time geometry. The antiparticles also carry spin. The positron is also a spin ½ particle. Antiparticles are also an inevitable consequence of space-time geometry when combined with quantum theory, which allows anything to happen that can possibly happen. An antiparticle is like a particle moving backwards in time. This is possible in quantum theory since the uncertainty principle allows for uncertainty in time as a consequence of uncertainty in energy. The positron is the spin ½ partner of the electron, which is like an electron moving backward in time. That movement backward in time gives the positron its positive charge.

The electron is a spin ½ particle. In the sense of dynamical variables represented by operators, electron spin is described by a Pauli spin matrix. The spin of the electron and the positron taken together as a holistic solution to the Dirac equation is described by the gamma matrices that Dirac discovered, which generalize the Pauli spin matrices to include the spin of antiparticles.

For the simpler Pauli spin matrices, the electron's spin angular momentum is described by spin eigenstates. An eigenstate is a solution of an eigenvalue equation. The electron's spin is represented by an eigenvector, which is a 1 by 2 matrix, while the spin matrix is a 2 by 2 matrix. The eigenvalue is just an ordinary number. The eigenvalue equation is written as $S\Psi=\lambda\Psi$, where S is the spin matrix, Ψ is the eigenvector and λ is the eigenvalue. For the Pauli spin matrices, there are only two eigenvalue solutions, which are spin ½ up and spin ½ down. These two solutions are written in terms of the eigenvalues as $\lambda=+\frac{1}{2}\hbar$ and $\lambda=-\frac{1}{2}\hbar$. The corresponding eigenvectors Ψ are written as a spin vector that either points up or a spin vector that points down.

$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$S_z \chi_\pm = \pm \frac{\hbar}{2} \chi_\pm$$

$$S_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Electron Spin States and Spin Eigenvalue Equation

In classical physics, angular momentum is a 3-component vector in space $L=(L_1,L_2,L_3)$ defined in terms of momentum and position. For example, when an electron orbits a proton at a radius r and moves with momentum p, orbital angular momentum is given in magnitude as L=pr if the orbit is circular, but more generally is given in terms of an x-y-z coordinate system, where the vector components are given as $L_1=p_2x_3-p_3x_2$ with cyclic permutations for the other components. In quantum theory, the position and momentum variables are represented by operators that give a commutation relation of the form $x_1p_1-p_1x_1=i\hbar$. The simplest way to represent the momentum operator is in terms of a derivative as $p_1=-i\hbar\Delta/\Delta x_1$ and similar expressions for other components. It is then easy to show the angular momentum operator satisfies $L_1L_2-L_2L_1=i\hbar L_3$ with similar cyclic permutations. This kind of commutation relation has a special significance in mathematics as it defines a Lie algebra and a corresponding rotation group. The quantum algebra of orbital angular momentum is then understood to represent rotations in 3-dimensional space.

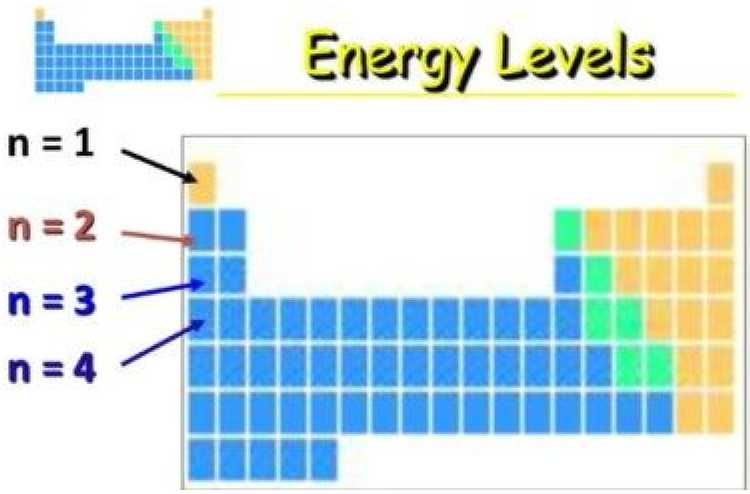
Electron spin is represented by the Pauli spin matrices as $S_1=\frac{1}{2}\hbar\boldsymbol{\sigma}_1$ and similar expressions. The commutation relations for these spin matrices satisfy the same exact commutation relations as the orbital angular momentum operators, namely $S_1S_2-S_2S_1=i\hbar S_3$ with similar permutations. Spin also represents rotations, but these are not rotations in space, but in space-time geometry. While the orbital angular momentum operators represent rotations in a spatial plane, like the x-y plane, the spin operators represent rotations in a spatial-temporal plane, like the x-t plane. This rotation group is similar to the rotation group of 3-dimensional space, but because of its special properties is called SU(2), which is visualized as rotations on the surface of a unit sphere.

Rotational symmetry plays an important role in physics, such as rotations around the unit circle in the complex plane, which is called a U(1) symmetry. The rotational symmetry represented by the orbital angular momentum operators in quantum theory is called an SO(3) symmetry. The rotational symmetry represented by the Pauli spin matrices is called an SU(2) symmetry. SU(2) symmetry is visualized as rotations on the surface of a unit sphere, which is a special kind of three dimensional rotation. The remarkable property of space-time geometry understood as a unified four dimensional geometry is both the symmetries of orbital angular momentum and the symmetries of spin angular momentum are inherent in the rotational symmetries of space-time geometry. The SU(2) symmetry of particle spin is inherent in the rotational symmetry of four dimensional space-time. There are actually two SU(2) symmetries inherent within space-time geometry. These two SU(2) symmetries describe spin ½ particles with different parities.

Parity is understood in physics as symmetry under mirror reflection, where right-handed turns into left-handed and left-handed turns into right-handed. The two SU(2) symmetries inherent in space-time geometry describe the parity of right-handed and left-handed spin ½ particles. This is visualized as the thumb pointing up when fingers of the left hand wrap around in a clockwise direction or when fingers of the right hand wrap around in a counterclockwise direction. This symmetry is the reason electron spin is described by a four component spinor wave-function. This description of particles in terms of right-handed and left-handed parity plays an important role in nuclear physics, where certain nuclear reactions violate parity conservation. In these nuclear reactions, the decay rate for right-handed particles is different than the decay rate for left-handed particles. This violation of parity conservation is an example of symmetry breaking.

Spin angular momentum has profound effects on atomic structure. We know from the solution to the hydrogen atom there are atomic orbitals characterized by discrete quantized energy levels. The lowest energy level is called the ground state. We might think that for an atom with many electrons we could fit all the electrons into the ground state orbital, but that's not correct. The Pauli exclusion principle says two electrons cannot occupy the same state. Since an electron has two possible spin states, either spin up or spin down, that means we can put at most two electrons into any quantum state. At most two electrons with opposite spin states can occupy the ground state, and then the next two electrons must occupy the next quantum state, and so on.

This way of progressively filling up quantum states with at most two electrons with opposite spin states per quantum state is the explanation for the periodic table. For any given un-ionized atom, the total number of electrons in that atom must equal the total number of protons in the atomic nucleus. The electrons must occupy progressively higher and higher energy levels since no more than two can occupy any quantum state. These atomic quantum states have complex orbital angular momentum states described by quantum numbers ℓ and m, which represent quantized values for the L^2 operator and the L_3 operator. The ℓ and m quantum numbers give rise to the periodic nature of the periodic table. Each row of the periodic table corresponds to exhausting all possible values of the ℓ and m orbital angular momentum states and the electron spin angular momentum states for a given value of n that characterizes one of the Bohr E_n energy levels.



Number	Symbol	Possible Values
Principal Quantum Number	n	$1, 2, 3, 4, \ldots$
Angular Momentum Quantum Number	ℓ	$0, 1, 2, 3, \ldots, (n-1)$
Magnetic Quantum Number	m_l	$-\ell, \ldots, -1, 0, 1, \ldots, \ell$
Spin Quantum Number	m_s	$+1/2, -1/2$

What gives rise to the Pauli exclusion principle? It turns out that only the spin ½ fermion matter particles like the electron obey the Pauli exclusion principle. The spin 1 force particles like the photon have no such restriction. Like virtually everything else we've discussed, the underlying reason for the exclusion principle has to do with the symmetry of space-time geometry, but this a weird kind of symmetry. In quantum field theory, the spin 1 force particles are described by vector fields that obey field equations, like Maxwell's equations for the photon. A vector field like the electromagnetic field is described by a four-vector that varies in space and time. The four vector has four components that correspond to the four dimensions of space-time. The numerical value of each component varies from one point in space-time to another point in space-time. The important point is each component of the four-vector for each point in space-time is an ordinary number. Ordinary numbers commute with each other in the sense AB=BA.

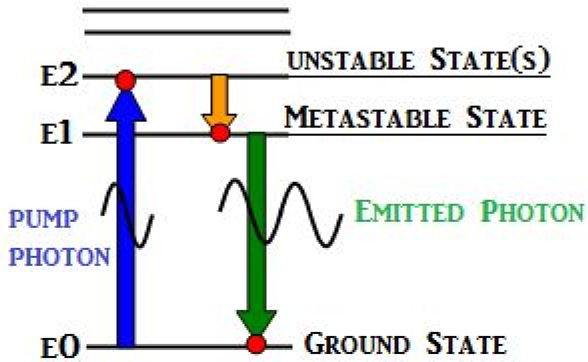
The spin ½ fermion particles are different. The spin ½ fermion particles are described by a spinor wave-function Ψ that also has four components like a four-vector, and each component varies from one point in space-time to another point in space-time. The spinor wave-function Ψ is different than a vector field A in that the spinors are described by anti-commuting numbers that obey a multiplication relation of the form $\Psi_1\Psi_2=-\Psi_2\Psi_1$ for two quantum states. The exclusion principle is a direct consequence of the spinor wave-functions as described by anti-commuting numbers, where $\Psi_1\Psi_2+\Psi_2\Psi_1=0$. Two spin ½ particles cannot occupy the same quantum state with the same quantum numbers since the square of an anti-commuting number is zero $\Psi^2=0$.

The essential difference between spin 1 particles like the photon, which are called bosons, and spin ½ particles like the electron, which are called fermions, is the bosons are described by wave-functions that are symmetric under interchange of the particles. This symmetry of the wave-function under interchange of the particles is represented by ordinary commuting numbers, for which AB=BA. The fermions are described by wave-functions that are antisymmetric under interchange of the particles. This antisymmetry of the fermion wave-function under interchange of the particles is represented by anti-commuting numbers, for which $\Psi_1\Psi_2=-\Psi_2\Psi_1$ and $\Psi^2=0$.

The Pauli exclusion principle along with quantum uncertainty explains why matter does not collapse. Quantum uncertainty is responsible for the orbital size of the hydrogen atom ground state. The uncertainty principle $\Delta p \Delta r \simeq \frac{1}{2}\hbar$ tells us that as uncertainty in the electron's orbital radius becomes smaller, uncertainty in its momentum becomes larger, and so the electron is forced to move away from the proton. This uncertainty gives rise to the Bohr radius of the ground state, for which $pr=\hbar$. The ground state is defined by fitting a single electron wavelength into the circumference of the orbit. In classical physics, the electron could radiate away all of its energy and collapse into the proton, but this is not allowed by quantum uncertainty.

The Pauli exclusion principle requires electrons to fill orbital states two by two, with each orbital state filled by two electrons with opposite spin states. Without the exclusion principle, all atomic electrons would eventually transition to the lowest energy ground state, and every atom would be no larger than the hydrogen atom. Without the uncertainty principle, the ground state would collapse to a singularity. Matter would collapse into a singularity without these two principles.

Unlike the spin ½ electron, where the Pauli exclusion principle tells us that two electrons cannot occupy the same quantum state, two spin 1 photons actually prefer to occupy the same quantum state. This effect is purely due to the photon wave-function being symmetric under interchange of the particles, while the electron wave-function is antisymmetric under interchange of the particles. This effect occurs in a laser, where the polarization states of the photons tend to align in the same direction of spin and the photon wave-functions also maintain a fixed phase relation, which results in the coherent light emitted by a laser. This coherence arises from a phase transition inside the laser, where all the atoms simultaneously make the same transition from an excited metastable state to the ground state. This effect is also the basis for a Bose-Einstein condensate when a massive number of identical particles occupy the same quantum state.

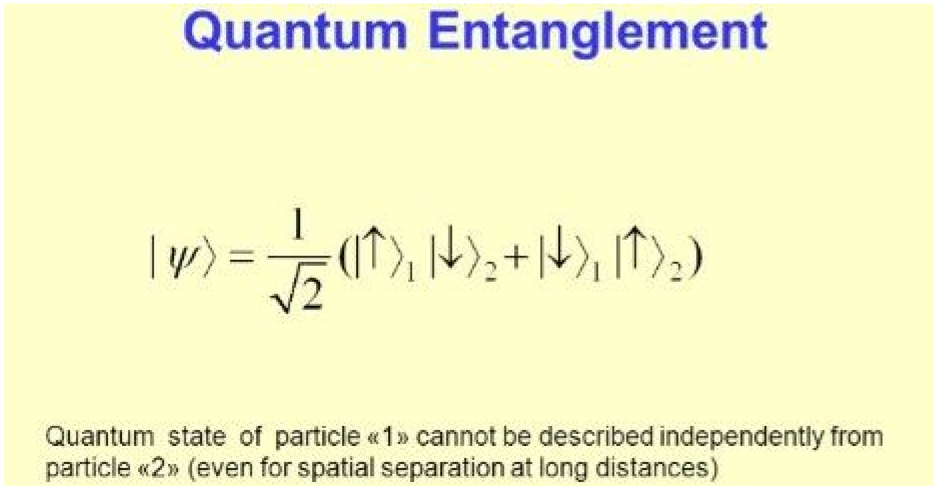


Where do the spinor wave-function anti-commuting numbers come from? The strange answer is they come from space-time. The most symmetric description of space-time geometry is when there are not only three spatial and one time dimensions, but when there is also another quantum dimension described by anti-commuting numbers. Once space-time has this extra symmetry, the exclusion principle is automatically in effect for the anti-commuting spinor wave-functions.

The vector fields of the spin 1 force particles and all other integer spin particles like the spin 2 graviton or the spin 0 Higgs particle are described by ordinary commuting numbers and so do not obey the exclusion principle. There is actually a tendency for the integer spin particles to collect together into the same quantum state. This collection into the same quantum state is an example of coherence, which is seen in the coherent light of a laser. Coherence for a wave-function just means the waves all tend to align with each other with the same phase relationship.

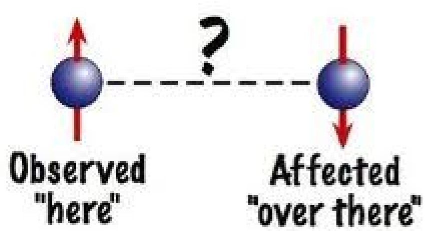
This tendency of the integer spin particles to become coherent is an example of entanglement. Quantum entanglement refers to a collection of particles that have entangled wave-functions. The wave-functions for all the photons in the coherent light of a laser are entangled. In the case of entangled photons this refers to a coherent relationship between the phases of all the waves. Entanglement implies that when the quantum state of one photon is measured, the quantum states of all the other entangled photons are also measured. The coherent light of a laser can result in the appearance of a coherent interference pattern on a photographic plate due to this tendency of all the entangled photons to become measured together. This effect is the underlying mechanism by which the coherent light of a laser turns a photographic plate into a hologram.

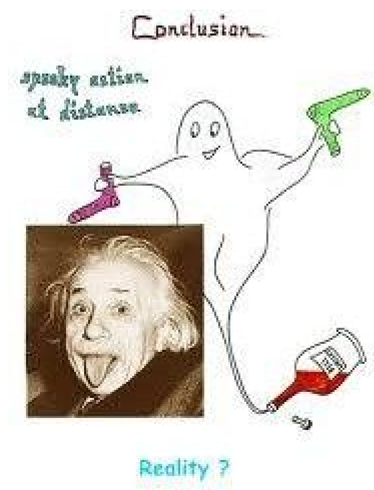
The classic thought experiment for entanglement is to consider the separation of two entangled particles. If a spin 0 particle like a meson decays into two spin ½ particles, those two particles are entangled since conservation of angular momentum requires one particle to be spin up if the other particle is spin down. The quantum state of this entangled pair has to describe all possibilities. If the first particle is spin down the second is spin up, and if the first is spin up the second is spin down. The quantum state is a sum over these two possibilities. The conventional way this quantum state is expressed is in bracket notation. The quantum state is written as a linear superposition of individual particle spin states as $\Psi=a[\uparrow>_1[\downarrow>_2+b[\downarrow>_1[\uparrow>_2$ where $[\uparrow>_1$ indicates particle 1 in a spin up state. This is the classic entangled wave-function.



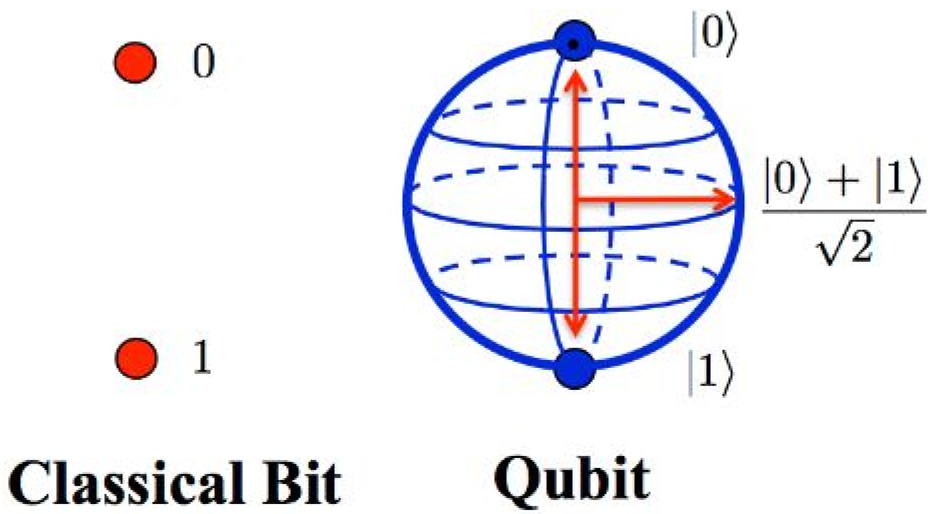
If the two particles move apart and separate, they remain entangled until measured. They could separate to opposite sides of the universe and remain entangled. No matter how far apart they separate, measurement of the spin state of one member of the pair instantaneously determines the measured spin state of the other member of the pair. In the sense of measurement as collapse of the wave-function, Ψ collapses to either $[\uparrow>_1[\downarrow>_2$ or $[\downarrow>_1[\uparrow>_2$ when the measurement is made. When the wave-function collapses for one particle, it also must collapse for the other particle.

This effect is so weird that Einstein called it spooky action at a distance. When two particles are entangled, measurement of the quantum state of one member of the pair instantaneously determines the measured quantum state of the other member of the pair no matter how far apart they separate. This tells us there is something weird about measurement. Since quantum entanglement is ubiquitous, measurement of entangled quantum states results in the nonlocal weirdness that a measurement over here affects another measurement over there.





Entangled spin states can also be represented in terms of bits of information, since a spin ½ state is like an on-off switch that encodes information in a binary code of 1's and 0's. A spin up state is like a 1 when the switch is on, and a spin down state is like a 0 when the switch is off. When a quantum state becomes entangled, that entangled state does not represent a classical bit of information, but rather a quantum bit of information that is called a qubit. A qubit is a quantum superposition over the spin up and the spin down states, and as such can represent something in between a 1 and a 0. Qubits are the basis for quantum information processing and computing.



Since a massless spin 1 photon can only have two polarization states that are described as either a right or a left circularly polarized state of spin angular momentum, the spin states of a photon can also encode bits of information in a binary code of 1's and 0's. Unlike the spin ½ electron that must obey the Pauli exclusion principle, spin 1 photons prefer to have their spin states align with each other when photons become entangled. This entanglement of photon spin states in the same direction of polarization results in the collective phenomena seen in a particle condensate.

Just like electron spin states, two circularly polarized photon spin states can be represented by a bit of information [1> or [0>. The weird thing about the entanglement of photon spin states is the alignment of the spin states allows for an entangled two photon state described as [00> or [11>, which are two photons that are circularly polarized in the same direction of spin. Entanglement of the spin 1 states allows for a superposition of these two states. These entangled photon states are written as Ψ=a[00>+b[11>. If the two photons separate, measurement of the spin state of one photon instantaneously determines the measured spin state of the other photon.

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

There is also something weird about the four-vector fields of the spin 1 force particles. These fields are described by another symmetry called gauge symmetry. The electromagnetic field is written in terms of a four-vector A that has four components $A=(A_0,A_1,A_2,A_3)$. This gauge field A(x,t) has the property that the electromagnetic force is invariant under a gauge transformation $A_a \rightarrow A_a + \Delta\Lambda/\Delta x_a$ where Λ=Λ(x,t) is an arbitrary function of space-time and $\Delta\Lambda/\Delta x_a$ indicates how this function changes for a small change Δx_a in the x_a-direction. In this notation, a=1,2,3 indicate the spatial directions x,y,z and a=0 indicates time as $x_0=ct$. This gauge symmetry is a freedom of the electromagnetic potential A(x,t) in that the electromagnetic force is invariant under this transformation and there is no locally measurable quantity that corresponds to the value of Λ. The invariant electromagnetic field F=F(x,t) is given in terms of the gauge potentials A(x,t) as $F_{ab}=\Delta A_a/\Delta x_b - \Delta A_b/\Delta x_a$ which is explicitly independent of the value of Λ.

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

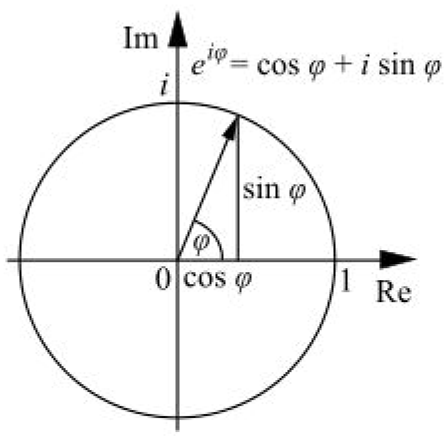
$$\partial_\mu F^{\mu\nu} = j^\nu$$

Maxwell's Equations of Electromagnetism

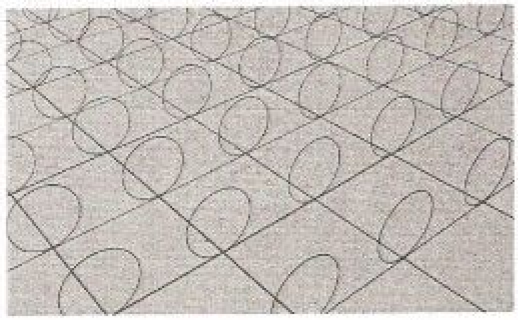
The reason gauge symmetry is important is because of the way the matter fields couple to the gauge fields. If we have a matter field described by a wave-function Ψ(x,t) that represents a matter particle like the spin ½ electron, the electromagnetic potential couples to the electron's wave-function in the electron's wave equation through a term that looks like $(\Delta/\Delta x_a - ieA_a)\Psi(x,t)$, where e is the charge of the electron and i is the imaginary number. This term seems to mix up the way the wave-function varies in space-time $\Delta\Psi/\Delta x_a$ with the electromagnetic potential A_a. This coupling is called a gauge interaction, which describes how electrically charged spin ½ matter particles interact with the electromagnetic field as represented by spin 1 force particles.

The gauge transformation $A_a \rightarrow A_a + \Delta\Lambda/\Delta x_a$ of the electromagnetic potential corresponds to a gauge transformation of the electron's wave-function $\Psi \rightarrow \exp(i\theta)\Psi$, where the angle $\theta = e\Lambda$ is now interpreted as a rotation in some internal space. This is weird. Gauge invariance is understood in terms of some angle of rotation θ that transforms the electron's wave-function as $\Psi \rightarrow \exp(i\theta)\Psi$.

The field equations for both the photon and the electron are invariant under this transformation, and therefore the laws of physics are invariant. $\theta(x,t)$ can vary from one point in space-time to another point in space-time in any arbitrary way, and the invariance is preserved. When the laws of physics are invariant, we say there is a symmetry that corresponds to a conservation law. This symmetry seems to be a symmetry under rotation by an angle θ in some internal space. Since the wave-function takes the form $\Psi(x,t) = \exp(i[kx - \omega t])$, the transformation $\Psi \rightarrow \exp(i\theta)\Psi$ results in a new wave-function $\Psi \rightarrow \exp(i[kx - \omega t + \theta])$ that only differs from the old one by a phase factor θ. Since $\exp(i\theta) = \cos\theta + i\sin\theta$, this phase factor is like a rotation in the complex plane by an angle θ.



The gauge symmetry looks like symmetry under rotation in some internal space. Since it can be described as a rotation in the complex plane, it's called a U(1) symmetry. The U refers to a unit vector in the complex plane, and the 1 refers to one direction of rotation. The other mathematical term for the four-vector A(x,t) is to call it a fiber bundle, which refers to a vector space that lives on top of space-time geometry. At every point in space-time, the four-vector A(x,t) has this gauge symmetry of rotation in some internal space. What exactly is this internal space? The answer given by the unification mechanisms of modern physics is this internal space is an extra fifth dimension curled-up into a small circle at every point of ordinary space-time geometry.



Compactified Extra Fifth Dimension Curled-up into a Small Circle

The Kaluza-Klein mechanism is a way of unifying gravity with electromagnetism. If we add an extra fifth dimension to our usual four dimensional space-time and curl up the fifth dimension into a small circle at every point of space-time, then Einstein's field equations for the space-time metric automatically give rise to Maxwell's field equations, where the electromagnetic field arises in terms of the extra components of the space-time metric with a fifth dimension. The extra fifth dimension is called a compactified dimension. It seems that modern physics wants to add extra dimensions all over the place, both in terms of extra compactified dimension and in terms of anti-commuting dimensions. The value of adding dimensions to space-time is in terms of extending the symmetries of space-time geometry. The Kalula-Klein mechanism of extra compactified spatial dimensions is a natural way of generating all the gauge symmetries and the addition of an anti-commuting dimension is a natural way of generating super-symmetry.

The transformation $x \rightarrow x+\Delta x$ is a translation in space. The conservation law that results from the invariance of the laws of physics under a translation in space is the conservation of momentum. This represents a symmetry of space-time geometry. The conservation law that results from gauge symmetry and the invariance of the laws of physics under a rotation in the internal space represented by the transformation $\Psi \rightarrow \exp(i\theta)\Psi$ is the conservation of charge. This is weird since the Kaluza-Klein mechanism tells us this is a rotation in an extra compactified dimension. The conservation of charge is the same as the conservation of momentum in the extra compactified dimension. As far as the Kaluza-Klein mechanism can say, particle charges like electric charge are ordinary spatial momentum quantized in a compactified dimension of space. This is weird since conservation of spin angular momentum results from invariance of the laws of physics under rotations in a space-time plane like the x-t plane. The factor $\exp(i\theta)$ represents a rotation in the complex plane. This generates the gauge symmetry of charge conservation that is understood as a symmetry under rotation in an extra compactified dimension of space. This symmetry underlies momentum conservation in that dimension, but is also like a rotation in a space-time plane that underlies the conservation of spin angular momentum. It seems as though a rotation in an extra compactified dimension of space is halfway between a translation in an ordinary extended dimension of space and a rotation in a space-time plane. This halfway between weirdness is represented by the factor exp(iθ) that generates a rotation in the complex plane.

This weirdness only gets weirder. Shortly after Einstein discovered gravity could be represented by field equations for the space-time metric, Hermann Weyl attempted to unify gravity with electromagnetism by postuating another symmetry that is now called conformal invariance. Weyl thought the laws of physics were invariant under a conformal transformation in which the space-time metric g scales as $g \rightarrow \lambda g$. The parameter λ is called a scale factor, which essentially rescales the length scale since the fundamental measure of length in space-time geometry is the proper-time interval. The proper-time interval in turn depends on the metric as $(\Delta\tau)^2=g_{00}(\Delta t)^2+...$

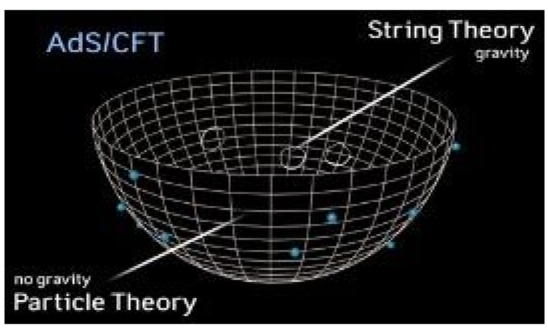
The metric g=g(x,t) is a 4x4 tensor matrix that in any given coordinate system (x,t) has sixteen values g_{ab} with a,b=0,1,2,3 that correspond to how distance is measured in a four dimensional curved space-time geometry. Measurement of proper-time is independent of whatever coordinate system we choose to measure things within, but our description of how we make those measurements can change as the coordinate system changes. We can always define a different system of clocks and rulers. When our (x,t) coordinate system description of space-time geometry changes, the sixteen values of the metric as specified in that coordinate system also change, but in such a way as to preserve the value of the proper-time interval $(\Delta\tau)^2=g_{00}(\Delta t)^2+...$ since proper-time is the only invariant measure of distance in space-time geometry.

Weyl wanted to rescale the space-time metric as $g \rightarrow \lambda g$, from which he was able to show Maxwell's equation were inherent in Einstein's equations. Ironically, he called this a gauge transformation referring to the gauge of railroad tracks. The problem is this transformation screws-up all measurements of distance in space-time geometry. Even the proper-time interval is no longer an invariant measure of distance in space-time if the space-time metric is rescaled along the lines Weyl proposed, except for the special case when proper-time is zero, where $\Delta\tau=0$ is invariant since zero can't be rescaled. The proposal was therefore abandoned, but a good idea can never really be killed. Soon after Weyl made his proposal, a different kind of rescaling was discovered to be at work in the quantum theory of gauge theories, like electromagnetism. The electron's wave-function was found to rescale under a gauge transformation as $\Psi \rightarrow \exp(i\theta)\Psi$, where the scale factor λ was given in terms of a unit-lengthed complex number that rotates in the complex plane $z=\exp(i\theta)=\cos\theta+i\sin\theta$. A gauge transformation is a special kind of rescaling.

In recent years, Weyl's idea of rescaling the space-time metric $g \rightarrow \lambda g$ has found a natural home in the idea of conformal field theories. What Weyl called a gauge transformation is now called a conformal transformation. A conformal field theory is invariant under conformal transformations in which the length scale is rescaled to a new scale. Invariance means the physics of the theory, which essentially is the interactions of the theory, does not change as the length scale is rescaled. If we think of some kind of fundamental interacting particles, and imagine that the particles bind together into larger composite particles, conformal invariance means the interactions between the composite particles look exactly like the interactions between the fundamental particles.

Conformal field theories are interesting because of the AdS/CFT correspondence. Anti-de Sitter space is a special kind of curved space-time geometry, which has a boundary called an event horizon. It is possible to define a conformal field theory on the event horizon of anti-de Sitter space. The weird thing that has been discovered is that the conformal field theory defined on the boundary is mathematically equivalent to a theory of gravity in the bounded space. This is weird since the bounded space has an extra dimension that the boundary does not have.

How can theories in different dimensions be mathematically equivalent? The answer is the holographic principle, which says everything that appears to happen in the bounded space is like a holographic projection from the boundary. The boundary acts as a holographic screen that projects holographic images into the bounded space, just like a hologram on the back of a credit card. The theory of gravity that appears to be in effect in the bounded space emerges from the more fundamental conformal field theory defined on the boundary. The mechanism by which the theory of gravity emerges in the bounded space is called thermodynamic emergence, since it arises from the thermodynamic properties of the boundary theory. Since the theory of gravity in the bounded region of space arises through holographic projection from the boundary theory, in some sense gravity is illusory, like the projected holographic images of a hologram.

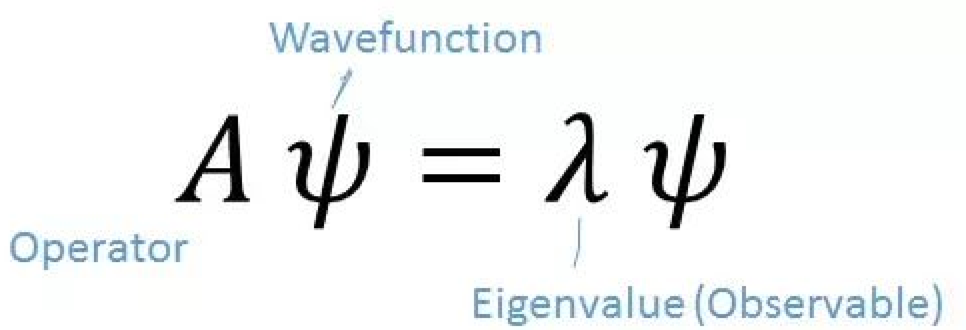


Where does the extra dimension come from? How can a theory defined on a boundary of space be equivalent to a theory with an extra dimension defined within the bounded region of space? The answer is the theory on the boundary has a kind of symmetry called conformal invariance that allows the length scale to be rescaled. This rescaling of length acts like another dimension.

The reason theories in different dimensions can be equivalent is because the extra dimension that characterizes the theory of gravity in the bounded space in some sense is an illusion. Not only is a theory of gravity in the bounded space emergent, but the space-time geometry of that bounded region of space is also emergent. The holographic principle is describing emergent space-time. The extra dimension of space that emerges through holographic projection is illusory in the exact mathematical sense that it only arises from the scale invariance of the conformal field theory.

Conformal invariance means the conformal field theory is invariant under a transformation that rescales the length scale. This mathematical transformation has all the properties of a physical dimension of space, such as measurement along a number line, a Pythagorean distance relation in which different dimensions are orthogonal to each other, and conservation laws that arise from invariance of the laws of physics with translations through space. The extra dimension of space comes from conformal invariance, which is a weird way to think about dimensions.

Although the idea of point particles with spin seems weird, since point particles by definition are dimensionless points and the concept of angular momentum carries with it the idea of some kind of rotation around an orbital radius, the idea of spin is perfectly consistent with the mathematics of space-time geometry and relativistic quantum theory. This mathematics is weird because it's expressed in terms of operators. The wave equation is really an operator equation, where ordinary dynamical variables like momentum are represented by differential operators and other dynamical variables like spin are represented by matrices. The wave equation is fundamentally defined by the Hamiltonian operator, which represents the total energy of whatever system of particles we are interested in, like the hydrogen atom. The way we solve for the measurable values of that system of interest, like the measurable quantized energy levels of the hydrogen atom, is to solve the wave equation, which is fundamentally written down as an eigenvalue equation in terms of the Hamiltonian operator operating on a wave-function.



The way the problem is solved in quantum theory is to write down an operator equation in which the Hamiltonian operator operates on some wave-function to result in the quantized energy levels of the system $H\Psi_n=E_n\Psi_n$. The parameter n is an integer that refers to one of the quantized energy levels. If the Hamiltonian operator consists of differential operators, this equation takes the form of a wave equation, like the Dirac equation, or the Schrodinger equation that is the non-relativistic limit of the Dirac equation. If the operators are matrix operators, then this is a matrix equation. In any case, when the Hamitonian operator H operates on one of the wave-functions Ψ_n labeled by the integer n, it gives back the quantized energy level E_n of that particular quantum state. This kind of operator equation is called an eigenvalue equation. The wave-functions Ψ_n are called eigenvectors and the energy levels E_n are called eigenvalues.

The first thing to be clear about is the wave-functions Ψ_n are not the quantum state of the system of interest. These wave-functions represent particular quantum states that have particular values of quantized energy. As Feynam tells us, the quantum state is defined as a sum over all possible states of the system. In order to construct the quantum state, we'd have to perform a sum over all the Ψ_n wave-functions. Each term in the sum would have a probability factor that would specify the probability with which we could measure that particular quantum state when we performed a measurement on the system. In general, the system has some probability of being in any possible quantum state, and we could measure any possible energy level when we measure the system.

The wave-functions Ψ_n are called eigenvectors because they form a vector space called Hilbert space. In the same sense the x-y-z coordinate system consists of three independent orthogonal directions of motion, the Ψ_n eigenvectors are also orthogonal to each other. If the integer n runs from 1 to N, this forms an N-dimensional vector space. If we think of each independent direction in this vector space as a number line, the probability factors that specify how likely the system can be measured in any one of these particular quantum states is like a distance along that number line, except in general the probability factors are complex numbers and so the number line is actually more like the complex plane. In any case, specifying all the probability factors determines a unique point in the N-dimensional vector space. This unique point specifies one possible quantum state of the system. The eigenvectors are not only orthogonal in the sense of independent directions of motion, but they are also complete in the sense they can describe any possible state of the system. Specifying a point in this vector space specifies a possible state.

The most general quantum state of the system is a sum over all the eigenvectors, each weighted with a probability factor. This determines a unique point in the vector space. When we measure a particular quantized energy level of the system, we are collapsing this point to a particular axis of the vector space. That is the meaning of the collapse of the wave-function. The most general wave-function Ψ is a sum over the Ψ_n eigenstates weighted with probability factors. When we measure a particular energy level E_n we are collapsing the wave-function Ψ onto one of these particular Ψ_n eigenstates. In the formalism of quantum theory, that is what a measurement does.

This conceptual formulation of quantum theory really has no idea how measurements occur. All it can say is that when a measurement occurs, the wave-function collapses, which is also called quantum state reduction. The probability factors that characterize the sum over all possible quantum states determine the probability with which measurements are made. However, there is a caveat. Collapse of the wave-function has to occur randomly for the laws of physics to have predictability. The quantum state is like a probability distribution. Only random measurements of that probability distribution give accurate measurements. If bias arises in the way measurements are made, the probability distribution is not accurately measured. In colloquial language, if bias arises in the way measurements are made, then all bets are off. The game is rigged. If bias arises in the way measurements are made, the laws of physics lose their predictability.

Feynman tells us the quantum state can always be formulated as a sum over all possible paths, where each path is weighted with a probability factor exp(iθ) that depends on the action. The laws of physics are inherent in the action principle. The classical laws of physics result from minimizing the action. Again, this way of formulating the quantum state is like probability distribution, where the action is determining the probability factors. The classical laws of physics only emerge from this probability distribution when the path of least action is the most likely path in the sense of quantum probability. This happens naturally when measurements are made in a random way, but when bias arises in the way measurements are made, all bets of off and the laws of physics lose their predictability. The game is rigged.

There is one last thing to say about the Hamiltonian operator. In some sense, the Hamiltonian formulation is the defining formulation of quantum theory. Feynman's sum over all possible paths formulation is called the Lagrangian formulation, which makes manifest the central role the actions plays. The Hamitonian formulation is inherently an operator formulation, which gives rise to the wave-function and the wave equation formulation in a more natural way.

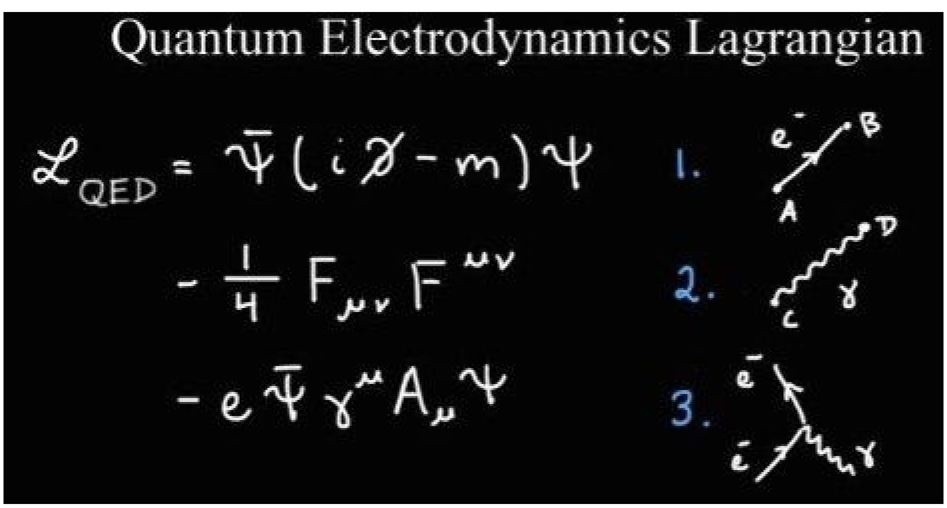
When the Hamiltonian operator operates on a wave-function, the result is time translation. This is mathematically expressed as $\exp(-iHT/\hbar)\Psi(x,t)=\Psi(x,t+T)$. The quantum operator $\exp(-iHT/\hbar)$ translates the wave-function forward in time by a time period T. This is called unitary time translation. The word unitary refers to the complex number $z=\exp(i\theta)=\cos\theta+i\sin\theta$ being a unit vector in the complex plane. This vector z makes an angle θ with the real axis of the complex plane and has a unit length. The Hamiltonian principle represented by this operator relation expresses energy conservation that arises from space-time symmetry and invariance of the laws of physics under translation through time. This idea of unitary time translation is as fundamental to quantum theory as Feynman's idea of the sum over all possible paths and the action principle. These ideas are actually equivalent since the Hamiltonian operator implies the action principle.

There is one weird last thing to say about energy conservation and invariance of the laws of physics under time translation. There is no such thing. The observable physical universe began at a point of singularity called the big bang event and will end in its heat death when the size of the universe expands to infinity. The singular big bang event is the beginning of time and the end of time is at infinity. There is no invariance under time translation because the universe is not at thermal equilibrium. The asymmetric direction of time, called time's arrow, is a result of the second law of thermodynamics, which tells us heat tends to flow from a hotter object to a colder object. Nothing can be hotter than the big bang event and nothing can be colder than the heat death of the universe. The observable physical universe is not invariant under time translation.

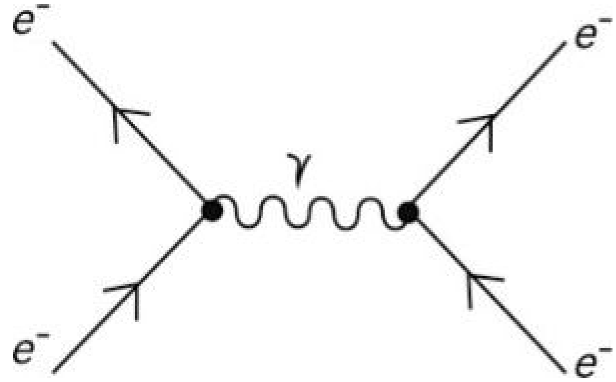
The observable physical universe is also not invariant under spatial translations. The universe is expanding in size at an accelerated rate, which tells us the size of the observable universe is limited by a cosmic horizon that limits the observations of an observer at the central point of view. There is really no such thing as invariance of the laws of physics under spatial translations and momentum conservation. At best, the symmetries of space-time geometry that give rise to the conservation laws of energy and momentum are idealizations that arise in some idealized limit when we ignore the asymmetrical nature of the observable universe. These idealizations may have practical value in terms of what can be measured in a physics lab, but do not apply to the asymmetrical nature of the observable universe. The whole concept of the symmetries of space-time geometry is flawed when we speak of the asymmetric observable physical universe.

Since the whole concept of space-time symmetry is flawed, the whole concept of quantum theory built on the conservation laws of that symmetry is also flawed. There must be something more fundamental than either space-time geometry or quantum theory that in some sense underlies these approximate idealizations. We'll soon discover with the holographic principle that what underlies these idealizations is the more fundamental concept of pure information. The way space-time geometry and quantum theory arise from pure information is called thermodynamic emergence. Both space-time geometry and quantum theory thermodynamically emerge from a more fundamental state of pure information that is described by the holographic principle.

That's basically all there is to quantum theory. The next step is just to work out all the details. Those details include the actual wave equations that wave-functions of specific particles obey. For example, the photon is the particle of electromagnetic radiation and its wave-function obeys Maxwell's field equations. The photon is coupled to the electron through the electromagnetic force, and the electron's wave-function obeys Dirac's field equations. When we put these field equations together and allow the particles to follow any possible path through space-time and to move in relativistic ways, we get the quantum field theory of quantum electrodynamics.



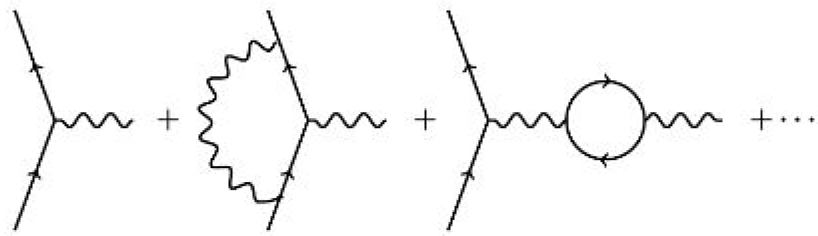
There are a number of odd details about quantum field theory that have to do with the nature of virtual particles and antiparticles. Uncertainty in energy allows virtual particles to be created along the lines of $\Delta E=\Delta mc^2$. Uncertainty in energy also allows virtual antiparticles to be created along the lines of $\Delta E\Delta t \simeq \frac{1}{2}\hbar$. An antiparticle is understood as a particle moving backwards in time, which becomes possible when time is uncertain. The quantum state is a sum over all possible particle paths through space-time, and so it is a sum over everything that can possibly happen, including the creation of virtual particles and antiparticles. This sum over everything that can possibly happen is represented by Feynman's sum over all possible paths, but it is also represented by Feynman diagrams. Feynman actually discovered the diagrams by examining the mathematical structure of the sum over all possible paths. To reiterate the point, the Feyman sum over all paths formulation of quantum theory is equivalent to the wave-function and wave equation formulation which is equivalent to the Feynman diagram formulation.



One of the great successes of quantum field theory is the calculation of the magnetic moment of the electron. Since the electron is a spin ½ particle that has a spin angular momentum of $\frac{1}{2}\hbar$ and also carries an electric charge of −e, it creates its own magnetic field which is called a magnetic moment. This is just like electrical currents inside the spinning earth or an electrically charged spinning top creating a magnetic field. The problem was when physicists calculated the electron's magnetic moment based on classical ideas of angular momentum, they got the wrong value. The calculated value disagreed with the measured value by a factor of 2, which is not a small error if you're trying to construct a fundamental theory of nature.

The solution had to wait for Dirac's discovery of the Dirac equation. The mathematics of gamma matrices in the Dirac equation supplied the extra factor of 2, and so all seemed to be well when the spin ½ electron was described by the Dirac equation. Dirac calculated the value of the electron's magnetic moment as $\mu=e\hbar/2m$. However, the nasty experimentalists continued to measure the electron's magnetic moment with greater precision and found another correction factor of 1.001159... The answer for this correction factor had to await the full formulation of quantum electrodynamics in which the effects of virtual particle-antiparticle pairs in Feynman diagrams were taken into account. These were corrections in terms of the fine structure constant

$\alpha = e^2/\hbar c \approx 1/137$. At the end of the day, the theoretically calculated and experimentally measured values for the electron's magnetic moment were found to agree to eleven significant figures.



Feynman Diagrams for the Electron's Magnetic Moment

The Higgs mechanism in quantum field theory is an example of spontaneous symmetry breaking, which is a way of understanding how complex behavior occurs in nature. The symmetry that is broken is the symmetry of space-time geometry, which is what gives rise to conservation laws. These conservation laws not only include the conservation of things like energy and momentum, but also things like spin angular momentum, which inherently mix up space with time.

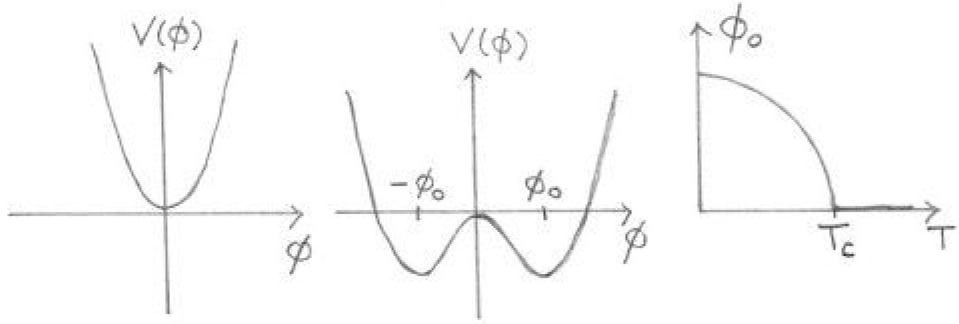
The classic example of spontaneous symmetry breaking is the spontaneous magnetization of a magnet. The magnetic material is composed of a bunch of atoms, which in turn are composed of electrons and atomic nuclei. The electrons and atomic nuclei have intrinsic magnetic moments, which are intrinsic magnetic fields that arise from their spin angular momentum and their electric charges, sort of like an electrically charged spinning top generates a magnetic field. The atom also generates a magnetic field from the orbital angular momentum of the charged electrons. These atomic magnetic fields tend to align because of the attractive magnetic force that arises as magnetic field lines line up, the same way bar magnets tend to align. This attraction between the intrinsic magnetic fields of the atoms is what allows a magnetic material to develop a global magnetic field, like the kind of magnetic field we see in a macroscopic bar magnetic. The development of a macroscopic magnetic field arises from the microscopic magnetic fields.

This tendency for the magnetic fields of individual atoms to align is opposed by the random thermal motion of the atoms, which tends to disrupt that alignment. There is always a balance between the potential energy of an attractive force that tends to cause alignment and the kinetic energy of random thermal motion that tends to disrupt that alignment. When there is too much random thermal energy at a microscopic level, which is characterized by a high temperature, no global alignment occurs and the magnetic material has no macroscopic magnetic field. When the temperature is low enough, the effects of potential energy dominate over the effects of random thermal motion and the magnetic material spontaneously develops a global magnetic field. The development of a macroscopic magnetic field arises from the microscopic magnetic fields when the microscopic tendency for alignment of atomic magnetic fields becomes greater than the microscopic tendency for disruption of that alignment that results from random thermal motion.

The way this is mathematically expressed is to write down a potential energy function. Since we are interested in magnets, let's call the magnetic field $\phi(x,t)$, which varies from one point in space to another point in space and also varies over time. For simplicity, we'll only consider one spatial dimension x. The potential energy function $V(\phi)$ is some function of the field. We don't know what form that function takes, but we can always expand it around some average value ϕ_0 and for small values of ϕ we can expand it in terms of powers of ϕ. We'll assume that only even powers of ϕ occur in the expansion in order to keep the potential energy positive for larger values of ϕ. This implies the potential energy function must have some minimum value, which by definition is its average value ϕ_0 that corresponds to the macroscopic value of the magnetic field.

The power series expansion for the potential energy function takes the form of a series of terms in powers of ϕ written as $V(\phi)=\phi_0+a\phi^2+b\phi^4+...$ We want to find the minimum value of this function, which is like looking for the bottom of a valley in a mountain range. That minimum value is the lowest energy state. We're looking for the lowest energy state just like we looked for the lowest energy state of the hydrogen atom. The way we find the lowest energy state is by seeing how $V(\phi)$ changes as we vary ϕ. The minimum of the valley will have a slope of zero, which corresponds to $\Delta V(\phi)=0$ for any variation in $\Delta\phi$. In terms of the power series expansion $\Delta V(\phi)=(2a\phi+4b\phi^3+...)\Delta\phi$. When we set $\Delta V(\phi)=0$, we get $(2a+4b\phi^2)\phi=0$.

There are two possible solutions to the equation $(2a+4b\phi^2)\phi=0$. The first solution is simply $\phi=0$. This is the correct solution when both a and b are positive parameters. The minimum value for $V(\phi)$ when both a and b are positive occurs at $\phi=0$. However, there is another possibility. The parameter a could be negative and the parameter b could be positive, in which case the minimum value does not occur at $\phi=0$. The potential energy function is no longer a single valley with its minimum at $\phi=0$, but instead has two adjacent valleys that occur for non-zero values of ϕ.



Spontaneous Symmetry Breaking

Let's examine this second case in greater detail. How could the value of the parameter a become negative? This is where the rubber meets the road. We have to make some kind of an assumption. Let's assume the parameter a depends on the temperature of the system. We know that our system of interest, which is some magnetic material, is characterized by a tendency for the atoms in that material to move around with random thermal motion, which gives rise to the temperature of the system. We also know the atomic magnetic fields of those atoms have some tendency to align due to the attractive force of magnetism. Let's assume the parameter a gives us a measure of the kinetic energy of random thermal motion, but also gives us some measure of the potential energy of the attractive magnetic force. When the temperature is too high and there is too much random thermal motion in the system, there is no macroscopic magnetic field, which corresponds to a positive value for the parameter a and $\Phi=0$. When the temperature is low enough, the parameter a turns negative and the minimum value occurs for a non-zero value of Φ.

Since we want to express this relation in terms of temperature, let's assume that near this turning point, the parameter a is a simple linear function of the temperature T. The simplest function we can write is $a=c(T-T_c)+\ldots$, where T_c is the temperature of the turning point. When T is greater than T_c the parameter a is positive, but when T is less than T_c the parameter a turns negative. The turning point $T=T_c$ is called the critical temperature at which the phase transition occurs. When T is less than T_c and the parameter a turns negative, the minimum value of $V(\Phi)$ occurs for $(2a+4b\Phi^2)=0$ which gives a non-zero value of $\Phi^2=-a/2b=c(T_c-T)/2b$. This gives a non-zero value for $\Phi_0=\pm((c/2b)(T_c-T))^{½}$. This non-zero value for Φ_0 corresponds to spontaneous magnetization of the magnet when the temperature is low enough, which is called a phase transition. The ± sign indicates the magnetization can occur in either the positive or negative x-direction. This simple analysis makes some concrete predictions. Near the phase transition when T is very close to T_c the magnetic field of the magnet Φ_0 behaves like $(T_c-T)^{½}$. This power law behavior is confirmed experimentally, but the exponent is not exactly ½ but more like 0.4.

In the quantum field theory description of elementary particles we are not really that interested in the spontaneous magnetization of magnets, so why all the fuss about phase transitions? The answer is the Higgs mechanism, which is exactly like the spontaneous magnetization of a magnet. The Higgs particle is a very special kind of particle. Unlike the spin ½ matter particles like the electron or the spin 1 force particles like the photon, the Higgs particle is a spin 0 particle. The basic idea is that when the universe was very hot, the Higgs particle acted like any other kind of particle with a potential energy function that was minimized at $\Phi=0$, which is called the ground state. However, when the universe cooled, the Higgs particle underwent a phase transition like the spontaneous magnetization of a magnet and developed a non-zero ground state value of $\Phi=\Phi_0$. This non-zero ground state value for the Higgs field has enormous implications.

The biggest implication is that other quantum fields like spin ½ matter fields and even some of the spin 1 force fields develop masses through the Higgs mechanism. These other fields are typically coupled to the Higgs field in their potential energy functions through some kind of an interaction that takes the mathematical form $\lambda\Phi\psi^2$. When the Higgs field takes on a non-zero ground state value $\Phi=\Phi_0$ this term looks like $m\psi^2$, where $m=\lambda\Phi_0$. Even if the particle described by the ψ field was intrinsically massless, that particle appears to become massive due to the Higgs mechanism. The basic idea is the masses of all particles arise through the Higgs mechanism. Mass is something that is emergent and does not really exist at a fundamental level. Mass is a thermodynamic property of nature that emerges when the temperature is low enough.

The irony is that once we understand the holographic principle, we'll see that even elementary particles are emergent and do not really exist at a fundamental level. Even the whole concept of space-time geometry is emergent and does not really exist at a fundamental level. The whole concept of quantum field theory as a description of fundamental particles is built on a house of cards. Even the concept of space-time geometry as described by Einstein's field equations for the space-time metric is built on a house of cards. That house of cards is thermodynamic energence. The question is what underlies this thermodynamic emergence? The answer we'll discover with the holographic principle is pure information. Theoretical physicists who worked in the area of quantum field theory as recently as twenty years ago would never have believed this, but most physicists who work in the area of quantum gravity today pretty much accept this is the truth.

We still have the nagging problem that space-time geometry is the stage upon which all the drama of quantum theory is performed. Particles follow paths through space-time. When we measure the position of a particle at some point in space and at some moment of time, that measurement is an event in space-time. The wave-function is only a probability amplitude that specifies the probability with which that event can be measured.

The nagging problem is Einstein's theory of general reality tells us that space-time geometry is the stage upon which the drama of a quantized space-time geometry is performed. Einstein's field equations for the space-time metric determine the curvature of that space-time geometry, but the space-time metric is also the wave-function for the motion of a particle we call the graviton that follows some path through that curved space-time geometry. Can we have it both ways? Is space-time geometry really a fundamental description of the physical world? Obviously not.

The Mathematical Structure of a Holographic World

Is space-time really fundamental or is there something more fundamental than space-time? We'll have to come back to this question later, but basically what the holographic principle is telling us is that space-time is a holographic illusion that results from holographic projection. The thing more fundamental than space-time is pure information. The holographic principle is all about describing the nature of the information that underlies the perception of space-time. The holographic principle says that information underlies all perceptions. That's why the concept of an observer in an accelerated reference frame is so important. The relativistic observer is observing those perceptions. The holographic principle is telling us that all perceptions in some sense are illusions that arise from holographic projection.

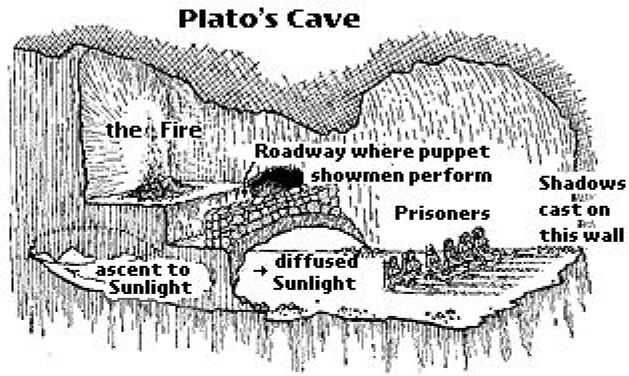
On the one hand, we have information that underlies all perceptions, and on the other hand, we have an observer that is observing those perceptions. The only thing that connects them is holographic projection. The perceptions in-and-of-themselves are illusory since they consist of nothing more than holographic projections of forms of information, like the images of a movie projected from a movie screen to an observer out in the movie audience. More fundamental than the illusory perceptions are the bits of information encoded on the screen and the observer out in the audience. The big question is about the true nature of the observer. This is a question physicists don't like to ask because it's a question about the nature of consciousness.

The holographic principle is a revolutionary idea that overthrows many concepts of modern physics, but not the equivalence principle, which underlies relativity theory, nor the uncertainty principle, which underlies quantum theory. Quantum theory is based on the uncertainty principle. The uncertainty principle says there are variables describing the nature of the world that cannot be simultaneously measured with perfect accuracy. For example, the position and momentum of a point particle cannot both be measured with perfect accuracy. You can measure one or the other but not both. The way this is formulated in quantum theory is to represent the dynamical variables with non-commuting variables. Non-commuting means the position variable times the momentum variable is not the same as the momentum variable times the position variable. With non-commuting variables, the order with which you multiply the variables matters and gives different results with different ordering. This is weird, but that is how quantum theory is formulated. The result is that you don't actually calculate what is observed to happen, but instead calculate a quantum state that describes everything that could possibly happen. This quantum state can be formulated in many equivalent ways such as the quantum wavefunction formulation, Feynman's sum over all possible paths formulation, and the formulation of vectors in Hilbert space. What all these formulations are describing are the quantum observables.

An observable is the value you actually measure when you go out and make an observation or do an experiment. For example, the vectors in Hilbert space are describing the observables, which are the values actually observed in a measurement. Quantum theory only allows you to calculate the quantum state. The quantum state can be understood as a sum over all possible observables. In effect, whenever an observation is made, one of these observables has to be chosen from the quantum state. The choice of an observable value goes by many names, such as collapse of the wavefunction or reduction of the quantum state. The quantum state is a sum over all possible observables and an observation is a choice of some actual observable value that's inherent in the quantum state. Implicit in this formulation is the idea of an observer observing the observable values. Relativity theory also has the implicit assumption within it that an observer is making an observation. The equivalence principle describes how observations are related when observers are in different accelerated reference frames. Quantum theory and relativity theory have nothing to say about the nature of the observer because physicists don't want to talk about consciousness, and yet here we are with both a relativistic observer and a quantum observer.

Can we put these two observers together and make them into one unified observer? Is there some connection between the relativistic observer in an accelerated reference frame and the quantum observer that is observing the observable values of the quantum state? The answer of course is yes. There has to be a way to put them together. This is exactly what the holographic principle accomplishes. That's why the holographic principle is the most fundamental principle, more fundamental that the uncertainty principle or the equivalence principle, but this unification comes at a price. The holographic principle is telling us that the price is neither quantum theory nor relativity theory can really be fundamental. Just like space-time is a holographic illusion that results from holographic projection, the quantum observables of the quantum state are also holographic illusions that result from holographic projection. In the strict mathematical sense of quantum theory, there is no such thing as a quantum observable. Such a thing is impossible for reasons we'll come back to later in this article. What we call observable values, like the position and the momentum of a particle, are just as illusory as the space-time geometry we observe.

It's time for a history lesson. We want to discuss the history of the holographic principle, which will tell us how we got here. Let's start the story 2400 years ago when Plato wrote the Allegory of the Cave. We'll actually take the story back quite a bit further to the story of Genesis, but let's start with Plato. Plato described a cave. The cave is a metaphor for the observable world. The cave has a wall. The wall is the boundary of the observable world. At this point it's not clear why the observable world should have a boundary, but for the moment let's just accept that it does. The wall of the cave acts as a screen, like a movie screen. Inside the cave there is a source of light that projects images on the wall of the cave, like a movie projector. Also inside the cave are observers observing the images projected on the wall of the cave, like observers out in a movie audience watching movie images projected from the screen. Plato called the observers prisoners because they're identifying themselves with the images they're observing. The observers have mistaken the images projected on the screen for themselves and don't know who they really are. They're confused about their identity. They've identified themselves with an image they perceive.



This sounds weird, but that's because the holographic principle is weird. The only way we can understand what Plato is saying is to understand the metaphor. The cave is the observable world and the wall of the cave is the boundary of the observable world. That boundary acts as a holographic screen that projects holographic images to the observers in the cave. Everything observable in the cave is a holographic image projected from the screen. What does that make the observers? Are the observers people? The answer is no. The observable image of a person is just another holographic image projected from the wall of the cave. Just like in relativity theory all we can say is the observer is a point of view that corresponds to an accelerated frame of reference. We'll come back to this later, but the wall of the cave is an event horizon that arises in the observer's accelerated frame of reference. Since nothing is observable beyond the event horizon, the event horizon defines the observer's observable world. It's the event horizon that acts as the holographic screen. The observer itself is only a point of view at the origin of a coordinate system that defines an accelerated frame of reference. If we want to get fancy about it, we can say the observer is the perceiving consciousness that is present at that point of view.

What about the images? Is there really a movie projector with film inside the projector? The answer is no. The projected images are forms of information. The holographic principle is telling us that the movie screen is really a holographic screen that encodes bits of information in some fundamental way. The holographic screen arises as an event horizon in the observer's accelerated frame of reference, and encodes bits of information due to some kind of fundamental encoding mechanism. We'll come back to this mechanism latter, but suffice it to say, it is a geometric mechanism. The projected images are forms of information organized on the holographic screen.

What about the light that is projecting the images on the wall of the cave. Is that physical light? The answer is no. To continue the metaphor, the projecting light is not physical light but the light of consciousness. We usually don't think about consciousness this way, but consciousness has both an outgoing projecting aspect and an incoming perceiving aspect. We call the perceiving aspect of consciousness the observer. The best name for the outgoing projecting aspect of consciousness is the light of consciousness. In a twisted way, the reason why the observers are identifying themselves with the images they're observing is because they're also projecting those images with their own light of consciousness. It's only a projection of their own bullshit.

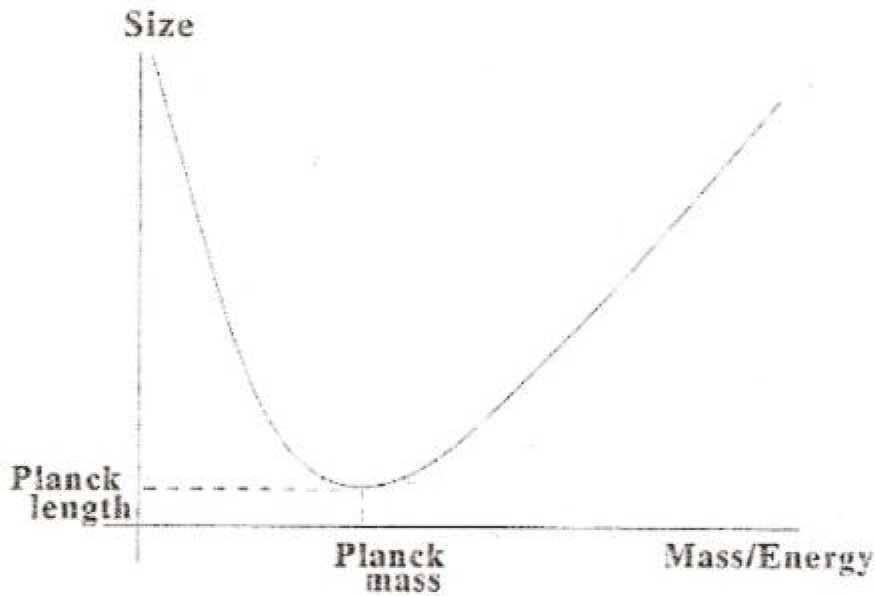
This bring us back to the story of Genesis, which just like Plato's cave is another metaphor for the holographic principle. Genesis tells us that in the beginning, God divided the light from the darkness. The light that Genesis refers to is the light of consciousness. Genesis also say the spirit of God moved over the face of the deep. The spirit of God is the observer in the sense of the perceiving consciousness, just as the projecting light is the light of consciousness. The source of both the perceiving consciousness and the light of consciousness is the darkness. The darkness is also consciousness, but it can't be described as either perceiving consciousness or the light of consciousness. It's the source of both. For lack of a better name, let's call it the source consciousness. When Genesis says God divided the light from the darkness, this implies the source is undivided. The source consciousness is one undivided consciousness. In order to create an observable world that is observed by an observer, the observer's perceiving consciousness and the projecting light of consciousness must be divided from the undivided source consciousness. In some sense, the observer's perceiving consciousness and the projecting light of consciousness are fragments of the undivided source consciousness. This act of division or fragmentation gives rise to individuality. Individual consciousness is always divided off from its undivided source. Individual consciousness has an outgoing projecting nature, which is the light of consciousness, and an incoming perceiving nature, which is the observer. The source of consciousness cannot be described in either of these ways because it is undivided. The undivided source of consciousness has no individuality, and has neither an outgoing projecting nor an incoming perceiving aspect.

When Genesis says the spirit of God moved over the face of the deep, this is a metaphor for the holographic principle. The spirit of God is the observer in an accelerated reference frame. The motion of the observer is its acceleration. The face of the deep is an event horizon that arises in the observer's accelerated frame of reference. The face of the deep is the wall of Plato's cave. The deep or the darkness or the abyss or the void is a way of describing the undivided source of consciousness in the sense of negation, or what it isn't. It isn't possible to describe what it is. Before the observer's observable world was created, there was only formless darkness or void. The observable world was only created because the consciousness of the observer was divided from the undivided source of consciousness, just like the light of consciousness was divided from the darkness. That observable world is always defined on a holographic screen, which is the face of the deep. The light of consciousness is projecting images of that world from the screen back to the observer, where the projected images are perceived. The whole thing is twisted since the observer can only perceive those images if its own light of consciousness projects them.

Let's pick up the story again in the 1960's. There were a number of physicists in the 1960's that realized the whole concept of quantum observables in quantum theory was flawed. In the strict mathematical sense of quantum theory, the concept of quantum observables is impossible. A quantum observable is something that in principle can be measured with exact precision. If we know absolutely nothing about a particle's momentum, in principle we can know everything about the particle's position. We can measure the particle's position with absolute certainty as long as its momentum remains totally uncertain. This is a direct consequence of the uncertainty principle. The problem is that when we add gravity to the equation the whole concept of quantum observables evaporates into the mist of holographic projection. Gravity throws a monkey wrench into the whole conceptual framework of quantum observables. That monkey wrench is called a black hole. Relativity theory tells us if we concentrate enough energy into a small enough region of space, we create a black hole. Once a black hole is created, nothing is observable beyond the event horizon of the black hole. If we pour more energy into that region of space, we just make the black hole bigger and even less is observable. The problem with quantum observables is that in order to measure something with greater and greater precision, we have to use a measuring device that concentrates more and more energy into a smaller and smaller region of space, which eventually leads to the formation of a black hole. The way this happens in practice is more accurate measuring devices use smaller and smaller wavelengths of radiation that carry larger and larger amounts of energy into smaller and smaller regions of space. Eventually we put so much energy into a small enough region of space that a black hole forms and nothing is observable beyond the event horizon of the black hole. Pour in more energy and the black hole's event horizon becomes bigger and even less is observable.

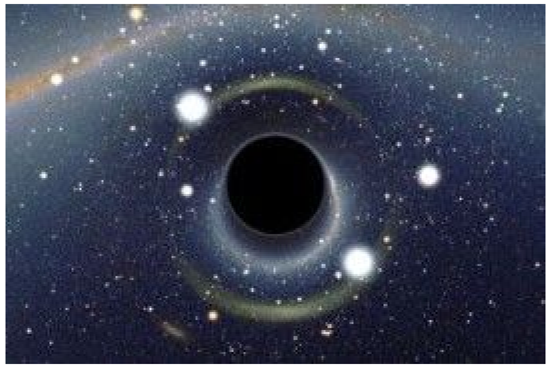
There is something fundamentally incompatible with the concept of quantum observables as formulated in quantum theory and the concept of gravity as formulated in relativity theory. The answer that physicists postualted in the 1960's was that observables can only be defined on a boundary at infinity. If we put measuring devices at infinity, they can measure observables with infinite precision. The concept of observables in any finite bounded region of space just does not make sense due to the problem of black holes, which will always throw a monkey wrench into the whole conceptual framework of observables. Observables simply do not exist in any finite region of space. Gravity does not allow exact quantum observables to locally exist in any finite region of space. Observables can only be defined on a boundary at infinity. The problem with this solution is infinity has no boundary. Once we put a boundary on infinity, it really isn't infinity any more. A boundary turns infinity into something finite. The real solution to this problem is to give up the concept of the existence of observables. What we call observables only have an illusory existence that arises from holographic projection. Observables do not really exist except as holographic projections of forms of information from a holographic screen to an observer at the central point of view, like the images of a movie projected from a screen.

A closely related problem to the incompatibility of gravity with the usual conceptual framework of observables in quantum theory is the problem of the concept of space-time. This is weird since the whole Einsteinian idea of relativity theory is that gravity is the curvature of space-time geometry. The problem again arises when we try to probe very small distances with very high energies. Since energy is equivalent to mass and mass gives rise to the force of gravity, if we concentrate enough energy into a small enough region of space, we create a black hole. The force of gravity is so strong at the event horizon of the black hole that even light cannot escape away from the black hole. Once we create a black hole, nothing is observable beyond the event horizon. Quantum theory tells us there is always some uncertainty in the amount of energy any measuring device can concentrate into a finite region of space due to the finite amount of time over which that energy is delivered. This uncertainty in energy tells us there is a smallest possible black hole where the uncertainty in energy and the uncertainty in size conspire to create a black hole of the smallest possible size. This tells us that there is a smallest possible distance scale that we can probe, which is the size of the smallest possible black hole.



Planck Length as the Smallest Possible Distance Scale

A black hole is a hole in space-time. A black hole forms whenever enough mass or energy is concentrated into a small enough region of space that the force of gravity within that region of space becomes so great that even light cannot escape from that region of space. This region of space is defined by the event horizon of the black hole. A light ray that originates inside the event horizon cannot escape from the black hole. This region of space is defined by solving Einstein's field equations for the space-time metric when there is a single gravitating object of mass M in that space-time geometry. The usual way to solve Einstein's field equations in this particular case is to solve them in the frame of reference of an accelerating external observer that remains stationary outside of the event horizon. This observer is accelerating since its reference frame is like an accelerating rocket-ship that uses the force of its thruster to remain stationary even though the force of gravity is pulling the rocker-ship toward the black hole.



As observed by the external accelerating observer that remains stationary to the event horizon, the solution to Einstein's field equations with a single gravitating mass M gives the proper-time interval as $(\Delta\tau)^2=(1-2GM/rc^2)(\Delta t)^2-(1-2GM/rc^2)^{-1}(\Delta r)^2/c^2$. The radial distance r is measured from the center of the black hole. There also are some angular terms, but we'll ignore them for simplicity. This is the famous Schwarzschild solution. The radius R of the event horizon is determined by $(1-2GM/rc^2)=0$, which gives $r=R=2GM/c^2$. As observed by the external observer, gravitational time dilation at the surface of the event horizon becomes infinite. As the accelerating external observer observes objects to fall into the black hole, it seems to take an infinite amount of time for those objects to cross the event horizon.

$$R=\frac{2GM}{c^2}$$

Schwarzschild Radius of a Black Hole Event Horizon

Gravitational time dilation is the effect that underlies all GPS navigation systems. If a satellite is in orbit a distance x above the surface of the earth, that satellite experiences an acceleration due to gravity as $a=GM/r^2$, where M is the mass of the earth, R is the radius of the earth and $r=R+x$. At the earth's surface the acceleration of gravity is $g=GM/R^2$. A clock on the surface of the earth appears to run slowly compared to a similar clock in the satellite as observed from the satellite. In the Schwarzschild solution, the effect of gravitational time dilation arises from the metric $(\Delta\tau)^2=(1-2GM/rc^2)(\Delta t)^2-(1-2GM/rc^2)^{-1}(\Delta r)^2/c^2$, which becomes infinite if $r=2GM/c^2$. We can use quantum theory as a trick to calculate this effect. If a photon is emitted from the earth's surface and rises to a height x above the surface, it loses an amount of energy $\Delta E=mgx$ that is converted into gravitational potential energy. The photon's effective mass is given by $E=mc^2$. The trick is to express this effective mass in terms of the frequency of oscillation of the photon.

Quantum theory tells us the photon's energy is related to its frequency as $E=hf$. This tells us the photon's effective mass is $m=hf/c^2$. The photon's energy therefore decreases by $\Delta E=hfgx/c^2$. This decreased energy gives a decreased frequency as $E'=E-\Delta E=hf(1-gx/c^2)=hf'$. The factors of Planck's constant cancel out for the same reason we don't see Planck's constant in Maxwell's equations. The frequency of the photon observed by an observer in the satellite is less than the emitted frequency by an amount $f'=f(1-gx/c^2)$. If the photon is emitted by an oscillating charged particle that oscillates with frequency f and if each oscillation of the particle is like the tick of a clock with a time interval Δt, the photon's frequency can be related to this time interval as $f=1/\Delta t$. As observed by an observer in the satellite, such a clock on the surface of the earth appears to run slower than a similar clock in the satellite since $\Delta t'=\Delta t/(1-gx/c^2)$. The effect of the gravitational field of a gravitating body is to slow down how fast clocks appear to run when clocks approach the gravitating body as observed by an observer from an accelerated stationary location in space. The ultimate effect of gravitational time dilation is to become infinite as time slows down to zero and appears to stand still at the event horizon of a black hole as observed by an external accelerating stationary observer in the space outside the event horizon. As observed by the accelerating external observer, objects seem to take an infinite amount of time to cross the event horizon as those objects appear to fall into the black hole.

On the other hand, for an observer in a freely falling frame of reference, which would occur if the accelerating external observer turns off the thrusters of its rocket-ship, falling objects are unhindered as they fall across the event horizon in a finite amount of time. Something weird however appears to happen from the perspective of an observer inside the event horizon. What was space for the accelerating external observer turns into time for the inside observer, and what was time for the external observer turns into space for the internal observer. Even weirder is the singularity at the center of the black hole, where time appears to come to an end and stop.

If there really is a smallest possible distance scale that any observer can measure, which is the size of the smallest possible black hole, then what do these strange happenings inside a black hole tell us about the nature of space-time geometry? How can space-time be a fundamental structure of the physical world if space-time geometry has holes in it within which time itself comes to an end at the central point of singularity of those holes in space-time? The answer as we'll soon see is that space-time geometry is not a fundamental structure of the physical world but is only a holographic projection of something more fundamental.

It's instructive to calculate the smallest possible distance scale that can be measured in terms of the size of the smallest possible black hole. The uncertainty principle tells us the uncertainty in the amount of energy measured is related to the uncertainty in the time of that measurement as $\Delta E\Delta t\approx\frac{1}{2}\hbar$. In a time interval Δt, light can travel a distance $\Delta x=c\Delta t$. Relativity theory tells us the radius of the event horizon of a black hole is given in terms of its mass a $R=2GM/c^2$. Let's call the size of the smallest possible black hole ℓ and write this smallest possible radius in terms of a smallest possible mass as $\ell=2G\Delta m/c^2$. Relativity also tells us this uncertainty in mass is related to the uncertainty in energy as $\Delta E=\Delta mc^2$. If we write $\ell=c\Delta t$ and put all these factors together we get $\ell=2G\Delta E/c^4\simeq\hbar G/\Delta tc^4\simeq\hbar G/\ell c^3$. We can then write $\ell^2\simeq\hbar G/c^3$.

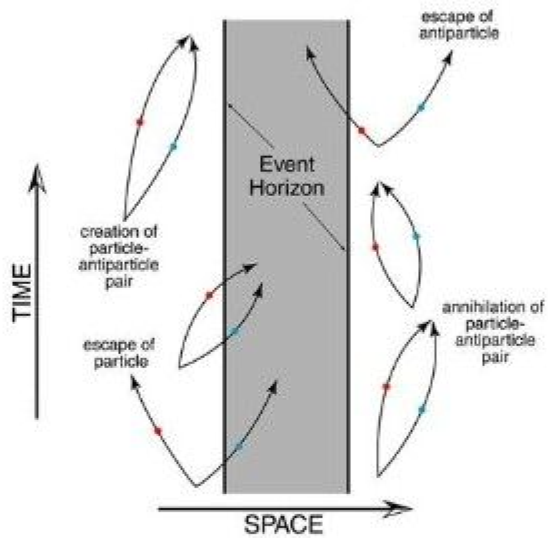
$$\ell_p = \sqrt{\frac{\hbar G}{c^3}} \sim 1.6 \times 10^{-35}\text{m}$$

This smallest possible distance scale is called the Planck length which is about 10^{-33} centimeters. The smallest possible black hole is called a Planck size black hole. Since nothing is observable smaller than this distance scale, the whole concept of space-time geometry as a continuum does not make any sense. The concept of a continuum is that of infinitesimally small points, like the points on a number line. If the number line is truly a continuum, it is possible to divide it up into smaller and smaller regions with the limit of infinitesimally small points. The problem is that space-time geometry cannot be infinitesimally divided up into smaller and smaller regions since there is a smallest possible distance scale that we can measure. It simply does not make any sense to talk about distances less than the Planck length. Space-time geometry is not a continuum. We could imagine that at the Planck length space-time has some sort of lattice structure, but that also does not make any sense. The only concept that does make any sense is the holographic principle, which tells us the space-time geometry we experience is a holographic projection from a holographic screen. The space-time geometry we experience is as illusory as the images of a movie we watch as images are projected to us from a movie screen.

We pick up the story again in the 1970's when there were two big developments. First, Jakob Bekenstein discovered that black holes have thermodynamic entropy. The concept of entropy has to do with the number of dynamical degrees of freedom any system can have. For example, a point particle has dynamical degrees of freedom in terms of its position in space and its velocity or momentum through space. These are the familiar dynamical variables we discussed in the context of the uncertainty principle and quantum theory. Bekenstein took a more idealized view of entropy as the bits of information that characterize specific objects or things. In this view, the dynamical degrees of freedom of any object or thing are these bits of information. A black hole is formed whenever enough things fall into a small enough region of space and enough mass or energy is concentrated into that region of space that the force of gravity within that region of space becomes so strong that even light cannot escape from that region of space. The attractive force of gravity acts on all matter and energy. Since light carries energy, which quantum theory tells us is proportional to the frequency of the light wave oscillations, and since that energy is equivalent to mass, the attractive force of gravity acts on light. The black hole is a region of space defined by its event horizon, which is a boundary in space that delineates a region of space from which even light cannot escape due to the attractive force of gravity.

In Bekenstein's view, the things that fell into the black hole were characterized by entropy, which he conceptualized as bits of information. Since all the things that fell into the black hole had entropy, the black hole also must have entropy. He was able to do an approximate calculation and discovered that the entropy of the black hole was proportional to the surface area of its event horizon. Within some numerical factor, the entropy of the black hole was equal to the surface area, A, of its event horizon divided by the Planck length, ℓ, squared. Since he conceptualized entropy in terms of bits of information, the total number of bits of information that characterized the black hole was approximately equal to the event horizon surface area divided by the Planck area, $n \simeq A/\ell^2$, where the Planck area is defined in terms of Planck's constant, the gravitational constant and the speed of light as $\ell^2 = \hbar G/c^3$. It was as if each Planck area on the surface of the event horizon acted like a pixel that encoded a bit of information. This discovery was very weird, since the things that fell into the black hole were inside the black hole, and if each thing carried a number of bits of information, the total number of bits of information that characterized the black hole should be proportional to the volume of the black hole and not to the surface area of the event horizon, but that's not what Bekenstein discovered. It was as though the event horizon of the black hole was like a computer screen and each Planck sized pixel on the screen encoded a bit of information. This discovery was the first indication of the holographic principle. As was eventually realized, the things that appeared to fall into the black hole were actually defined by bits of information encoded on a holographic screen. The appearance of things falling into the black hole was only a holographic projection from the holographic screen.

When Stephen Hawking heard about Bekenstein's discovery he couldn't believe it and he set out to disprove it. Much to his surprise, he ended up actually confirming it. Hawking was able to shore up Bekenstein's calculation and make it exact. Hawking examined how a quantum field behaved in the vicinity of the event horizon of a black hole. Quantum fields have the strange property that they can create virtual particle-antiparticle pairs everywhere in empty space. What Hawking heuristically discovered was that from the point of view of an external observer, virtual particle-antiparticle pairs could separate at the event horizon. The virtual antiparticle could fall into the black hole and disappear inside the event horizon while the virtual particle could appear to move away from the event horizon toward the external observer and appear to become a real particle. As far as the external observer could see, the event horizon of the black hole appeared to radiate away real particles. Hawking calculated the properties of this radiation and discovered it was a kind of thermal blackbody radiation. Blackbody radiation is the kind of thermal radiation that all hot objects emit due to the thermal motion of their atoms and molecules.

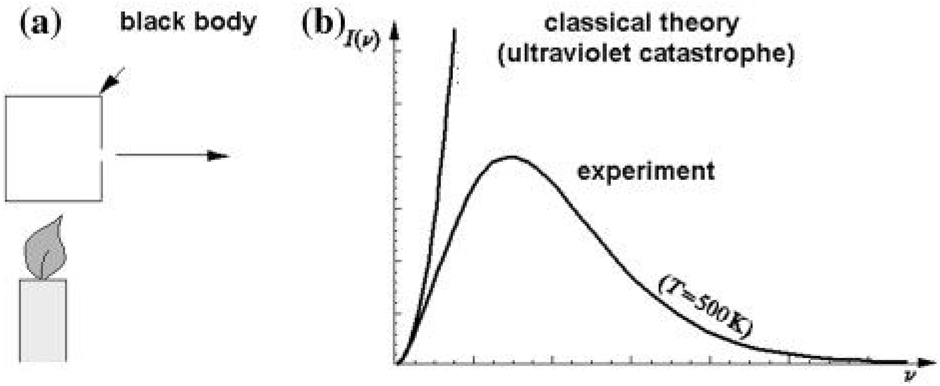


Hawking Radiation

Ironically, the whole field of quantum theory was first discovered by Max Planck when he tried to calculate the characteristics of blackbody radiation. Blackbody radiation is an everyday experience. It is the heat you feel when you open up a hot oven. That heat is in the form of infrared frequency electromagnetic radiation. That radiation arises from the thermal motion of atoms inside the hot oven. Planck discovered the atoms inside objects could not continuously emit radiation due to their thermal motion the way classical physics assumed, but instead had to emit radiation in discrete quantities that were eventually called quanta. Atoms inside objects move around because they have a kind of kinetic energy that is proportional to the temperature of the object. This thermal energy is understood as randomized kinetic energy. Each independent direction of motion is another degree of freedom. Ludwig Boltzmann discovered that this kinetic energy of thermal motion could actually define absolute temperature as E=kT.

The atoms inside objects do not move around freely, but are bound together in some sort of aggregate, like a crystal. Instead of freely moving around, the atoms tend to oscillate around some fixed position. Like the strings of a guitar, this kind of oscillation is called a harmonic oscillator. In classical physics, an electrically charged harmonic oscillator can continuously emit electromagnetic radiation with the frequency of radiation proportional to the frequency of oscillation. Planck assumed the blackbody radiation of a hot object was due to the thermal oscillations of atomic harmonic oscillators inside the object. When he performed his calculation assuming the continuous emission of radiation, he ran into a little problem. The total amount of energy emitted as blackbody radiation from a hot object was infinite. This calculation implied that when you open up a hot oven and look inside, your head should melt. The only way Planck could make the result of the calculation finite was if he assumed the atomic harmonic oscillators do not emit radiation continuously, but rather in discrete quantities that are now called quanta.

Planck discovered the emission of radiation from an atomic harmonic oscillator was quantized in terms of the famous formula for quantized energy as E=hf, where h is Planck's constant and f is the frequency of oscillation. His calculation gave a perfect fit to the experimentally measured spectra of blackbody radiation, so perfect that is was possible to determine the value of h to a high degree of precision. Once this discovery was made, quantum theory became inevitable.



Blackbody Radiation and the Ultraviolet Catastrophe

It's instructive to repeat Planck's calculation. Planck understood a hot blackbody in terms of atomic theory as a collection of atoms that behaved like little harmonic oscillators. The atoms carried electric charges in the form of electrons, and so as they oscillated with some inherent frequency ω, they emitted electromagnetic radiation of the same frequency, much like the oscillating electrons in a radio antenna. The only difficult part of the calculation was to calculate how the atoms oscillated at a temperature T. At thermal equilibrium, which is characteristic of a hot blackbody for which the emission and absorption of thermal radiation are in equilibrium, each dynamical degree of freedom of the system carries an average thermal energy E=kT. These dynamical degrees of freedom are the oscillations of the atoms around some equilibrium position, like the oscillations of a pendulum or the oscillations of a guitar string. Each such atomic oscillator has its own inherent frequency of oscillation ω that depends on a number of factors such as how the atom binds to other atoms. Each such atomic oscillator can emit radiation at the same frequency with which it oscillates. The spectrum of blackbody radiation from this collection of atoms depends on the distribution of atomic oscillators and how they oscillate at thermal equilibrium with average thermal energy E=kT.

The basic problem was to calculate how the oscillator oscillates at thermal equilibrium. This problem had been addressed by Boltzmann in his exploration of the kinetic theory of gases. Boltzmann understood that a moving particle could move in any possible way, but there was a probability distribution at thermal equilibrium that described the probability of any possible motion. This probability distribution is characterized by a probability factor that is called the Boltzmann factor. This probability factor is an exponential function of the total energy E of the system of interest when all the particles in the system move in some particular way. This probability factor is written as $P=\exp(-E/kT)$. This probability factor should look familiar since it's similar to the probability factor in the Feynman sum over all possible paths formulation of quantum theory, which is written as an exponential function in terms of the action.

Planck used this probability factor to calculate the average thermal energy of a collection of atomic oscillators at temperature T and found the average thermal energy of each oscillator was simply $\langle E\rangle=kT$. The average energy $\langle E\rangle$ is just a sum over all possible values of energy E weighted with the probability factor $P=\exp(-E/kT)$. The sum is over all possible energies without restriction. This result is a problem since the amount of thermal radiation emitted from a hot blackbody also depends on factors of the frequency ω of oscillation, as the spectral intensity of radiation behaves like $I=\omega^2\langle E\rangle/c^3$. This implies more radiation is emitted at higher frequencies. A hot blackbody should radiate more thermal radiation the higher the frequency of radiation without limit. If we include all possible frequencies of radiation like x-rays and gamma-rays, there is no limit to how much thermal radiation can be radiated away. Look inside a hot oven and the x-rays and gamma-rays should melt your head away.

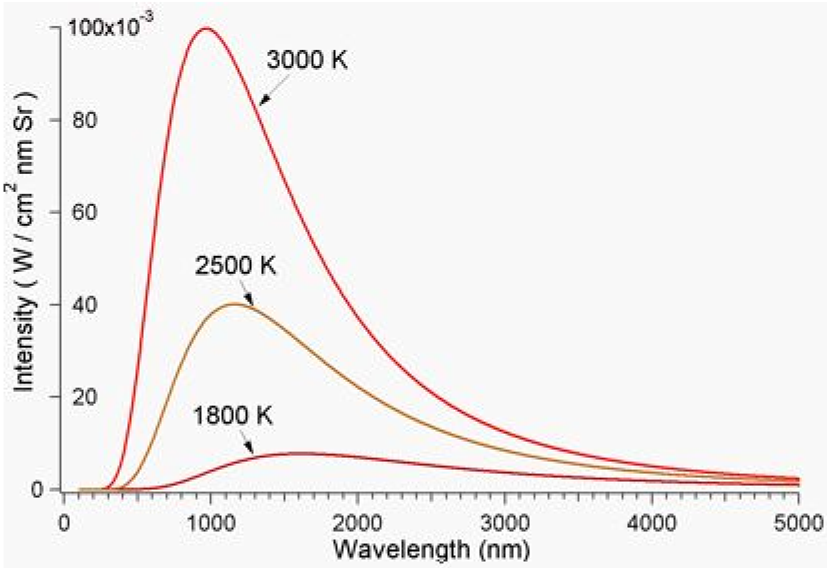
This obviously is not correct. Planck made an inspired guess the atomic oscillators were limited in the way they could oscillate. He did not understand this limitation, but just assumed there was a limitation in the way the oscillators could oscillate and repeated the calculation. The limitation he assumed was in terms of how energy is related to the frequency of oscillation.

Planck assumed the oscillators could not oscillate with any energy of oscillation but only with certain restricted energies of oscillation. This was weird since in classical physics the energy of oscillation is proportional to the amplitude of oscillation, but instead Planck assumed the energy of oscillation was proportional to the frequency of oscillation. Planck wrote these permitted energies as $E_n=n\hbar\omega$ in terms of an integer $n=0,1,2,3,...$

This assumption meant that Planck had to make a sum over discrete values of energy weighted with the probability factors rather than a continuous sum over energy. The weighted sum over all energies takes the form $\langle E\rangle=\Sigma E_n\exp(-E_n/kT)$. If we call the probability factor $x=\exp(-\hbar\omega/kT)$, then due to the property of the exponential function $x^n=\exp(-n\hbar\omega/kT)$. In terms of the factor x, the sum is $\langle E\rangle=\hbar\omega[x+2x^2+3x^3+...+nx^n+...]$. This is actually an easy sum to perform.

Consider the sum $S=[1+x+x^2+x^3+...+x^n+...]$. Let's guess that $S=1/(1-x)$. If this is the correct guess, then $(1-x)S=1$, which can be confirmed term by term in the multiplication. In the same way, for the sum $S'=[1+2x+3x^2+4x^3+...+(n+1)x^n+...]$, we can guess $S'=1/(1-x)^2$, and confirm it term by term is the multiplication $(1-x)^2S'=1$. We can then write $\langle E\rangle=\hbar\omega xS'=\hbar\omega x/(1-x)^2$. We need to normalize this sum by dividing it by $\Sigma\exp(-E_n/kT)=[1+x+x^2+x^3+...+x^n+...]=S=1/(1-x)$.

The normalized sum for the average thermal energy is then simply <E>=ħωx/(1−x). Putting this back into the original form <E>=ħωexp(−ħω/kT)/(1−exp(−ħω/kT)). This is the final result that explains the thermal spectrum of blackbody radiation. This thermal spectrum has been measured in numerous situations in nature, from hot ovens to the thermal cosmic microwave background radiation left over from the big bang event. Everytime the spectrum of thermal blackbody radiation is measured, it takes this form. This is also how Planck's constant was first measured.



Blackbody Radiation

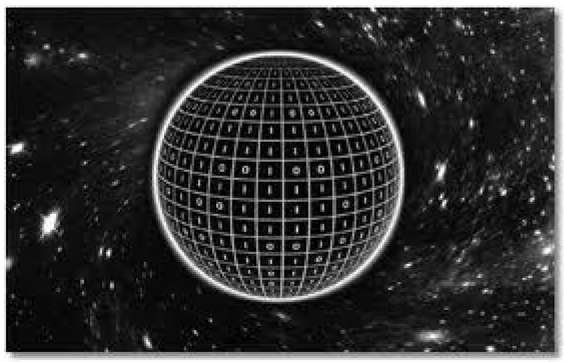
What about the classical calculation that resulted in <E>=kT. This result is the limit if we take Planck's constant to zero, since the leading term of 1−exp(−ħω/kT)=ħω/kT. The classical limit is for small energies where we can ignore Planck's constant. For large energies we need to use the full quantum form. In the limit of large energies, the spectrum of thermal blackbody radiation behaves like <E>=ħωexp(−ħω/kT) with an exponential cutoff for high frequencies.

Planck had no explanation for why the energy levels of an atomic harmonic oscillator should behave like E_n=nħω. That explanation would have to wait for a full quantum theory in which the quantized energy levels of an atomic harmonic oscillator can be solved with the wave equation.

The irony of Planck's discovery of quantum theory through this understanding of the nature of blackbody radiation was Hawking's discovery of the blackbody radiation from a black hole that eventually led to the discovery of the holographic principle. Hawking discovered the radiation emitted from the event horizon of a black hole as observed by an external observer was a kind of thermal blackbody radiation, no different in kind than the heat emitted from a hot oven. Hawking calculated the temperature of the black hole's event horizon and found kT=ħc/4πR, where R is the radius of the event horizon and is given in terms of the mass of the black hole as $R=2GM/c^2$.

This result can also be expressed in terms of the acceleration of gravity at the event horizon $g=GM/R^2$ as $kT=\hbar g/2\pi c$. Hawking used this temperature and the second law of thermodynamics to determine the black hole's entropy.

The second law of thermodynamics is quite simple. The second law simply says that when an object is at thermal equilibrium, the total amount of thermal energy in the object, Q, is the average amount of thermal energy per degree of freedom multiplied by the total number, n, of degrees of freedom. Boltzmann tells us the average amount of thermal energy per degree of freedom is given in terms of the temperature of the object as $E=kT$. Bekenstein conceptualized that thermodynamic degrees of freedom are represented by bits of information. In terms of these concepts, $Q=nkT$. Defining entropy as $S=kn$, then $\Delta Q=T\Delta S$. Hawking used his formula for the temperature of the event horizon and calculated the number of bits of information. Like Bekenstein, he discovered the entropy of the black hole was proportional to the surface area of the event horizon as $n=A/4\ell^2$. Unlike Bekenstein's calculation, this was an exact calculation.



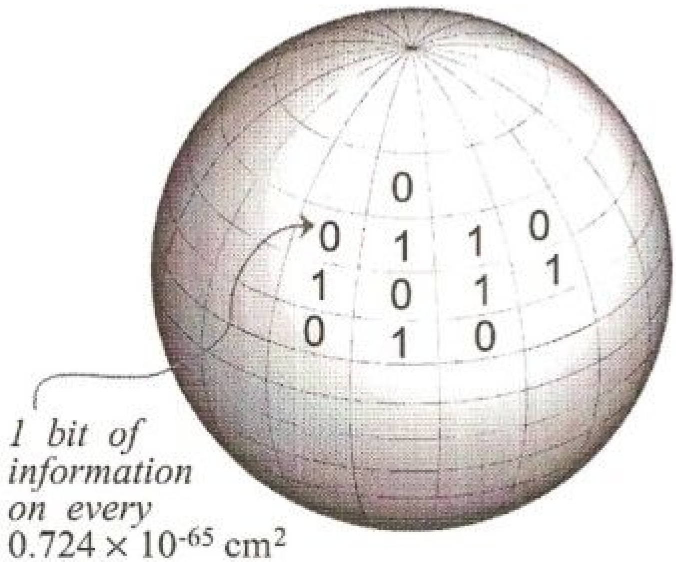
$$S_{\mathrm{BH}} = \frac{kA}{4\ell_{\mathrm{P}}^2}$$

Black Hole Entropy

It's instructive to perform this calculation heuristically in order to see what assumptions are involved in the calculation. The first thing is to define the black hole. Newton's law of gravity tells us that a mass creates a gravitational field, where the acceleration due to gravity is given in terms of the object's mass, M, and the distance R from the object as $g=GM/R^2$. Einstein's field equations for the space-time metric say basically the same thing. If a smaller mass, m, moves in that gravitational field, it will experience a force $F=mg$. Instead of discussing this gravitational force, it's more instructive to discuss the gravitational potential energy that underlies the force. The gravitational potential energy of the smaller mass moving in the gravitational field of the larger mass is given as $PE=-GmM/R$. The minus sign indicates gravity is an attractive force. The smaller mass moves around in space with a velocity, v, which gives that mass a kinetic energy of $KE=\frac{1}{2}mv^2$. The total energy of the smaller mass is then $E=KE+PE=\frac{1}{2}mv^2-GmM/R$. As the smaller mass moves around the larger mass, if it doesn't have enough kinetic energy to escape from the larger mass, it becomes bound to the larger mass and will undergo some sort of orbit around the larger mass. The total energy is negative since gravity dominates. If the smaller mass does have enough kinetic energy to escape from the larger mass, it is unbound and escapes away. The total energy is positive since kinetic energy dominates over gravity. The minimum amount of kinetic energy that corresponds to escaping away determines an escape velocity. This is the velocity a rocket-ship must achieve to escape away from the gravitational attraction of the earth. Escape velocity gives the smaller mass the minimum kinetic energy needed to escape from the larger mass. At any distance of separation, R, escape velocity is determined when the total energy of the smaller mass is zero. Setting $E=0$ then determines escape velocity as $v^2=2GM/R$.

The nature of a black hole is a region of space delineated by an event horizon where the force of gravity at the horizon is so strong that even light cannot escape. In some heuristic sense, escape velocity at the surface of the event horizon is the speed of light. If we set $v=c$ in the above formula for escape velocity, then the radius of the event horizon is determined as $R=2GM/c^2$. This result is confirmed by solving Einstein's field equations for the space-time metric. The Schwarzschild solution confirms this is the radius of the black hole's event horizon. This solution is only valid in terms of the observations made of the black hole by an external observer.

The next step in the calculation is to assume the event horizon radiates Hawking radiation as observed by an external observer. We know from quantum theory the energy of radiation is quantized in terms of its frequency as $E=hf$. The frequency of radiation is given in terms of its wavelength as $f=c/\lambda$. We have to make a guess about the wavelength of Hawking radiation. At the event horizon, the force of gravity is so strong that even light cannot escape. A light wave that is just barely bound to the black hole must have a wavelength that is approximately equal to the maximal circumference of the event horizon, which is $2\pi R$. If we set $\lambda=2\pi R$, the energy of a single particle of Hawking radiation is $E=hc/2\pi R$. As the black hole radiates away this radiation, it radiates away some of its energy. Since mass and energy are equivalent as $E=mc^2$, the amount of mass the black hole radiates away with Hawking radiation is $m=h/2\pi cR$. In other words, as the black hole radiates Hawking radiation, its mass decreases as $\Delta M=m=h/2\pi cR$. But we know what R is in terms of M, $R=2GM/c^2$, which tells us $\Delta R=2G\Delta M/c^2=hG/\pi Rc^3$. We can rewrite this result as $R\Delta R=hG/\pi c^3$. We've seen something like this before. The Planck area is $\ell^2=\hbar G/c^3$, and so we can write $R\Delta R=2\ell^2$. This is an amazing result. As Hawking radiation is radiated away from the event horizon of the black hole, the radius of the event horizon decreases in proportion to the Planck area. Actually, it's not the radius but the surface area of the event horizon that decreases in proportion to the Planck area. The surface area is given as $A=4\pi R^2$, and so a small decrease in surface area is given as $\Delta A=8\pi R\Delta R=16\pi\ell^2$. As each particle of Hawking radiation is radiated away from the event horizon, the surface area of the event horizon decreases by about a Planck area. If we imagine that each Planck area encodes a bit of information, then as each particle of Hawking radiation is radiated away from the event horizon, the surface area of the event horizon loses a Planck area and therefore loses a bit of information. Each Planck area on the surface of the event horizon encodes a bit of information that corresponds to the radiation of a single particle of Hawking radiation. The total number of bits of information encoded on the horizon is approximately equal to the surface area of the event horizon divided by the Planck area, just as Bekenstein discovered and Hawking confirmed is given by $n=A/4\ell^2$.



Holographic Principle

The reader should note the above calculation is essentially the same we previously performed to calculate the size of the smallest possible black hole. To say the event horizon of a black hole encodes bits of information in terms of its surface area as one bit of information per Planck area is the same as to say the smallest distance scale we can measure is the Planck length, since a Planck sized black hole must form whenever we probe that distance scale. The connection between these two findings is the nature of holographic projection from an event horizon.

The next step is to calculate the temperature of the event horizon. This is easy to do if we use the Boltzmann formula for the average thermal energy $E=kT$ of each degree of freedom and set this equal to the energy of a single particle of Hawking radiation. Each particle of Hawking radiation corresponds to a bit of information encoded on the surface of the event horizon. Each bit of information is a degree of freedom, and the average thermal energy per degree of freedom at thermal equilibrium is $E=kT$, which we set equal to the quantized energy of a single particle of Hawking radiation given by $E=hf=hc/\lambda=hc/2\pi R$, where we've assumed each particle of Hawking radiation has a wavelength approximately equal to the maximal circumference of the event horizon. This gives a result that looks very similar to the result Hawking found $kT=\hbar c/4\pi R$.

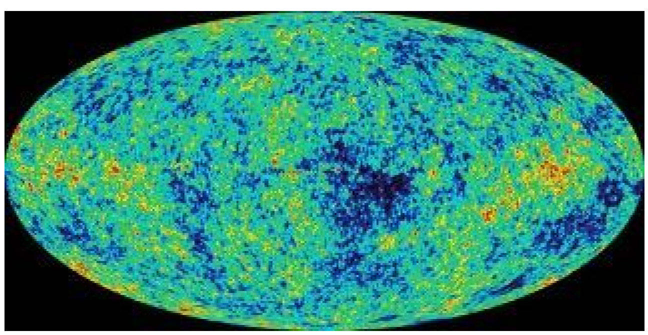
Shortly after Hawking made his discovery, he began to wonder about what happened to all the information that accumulated inside the black hole when things fell into the black hole. Those things carried entropy with them as they fell into the black hole and so the black hole accumulated all that entropy. The problem was that nothing can escape from a black hole and so that entropy or information must forever be trapped inside the black hole, or so it world seem. This was a problem because the black hole has a temperature and radiates away Hawking radiation. If the temperature of the surrounding space is colder than the temperature of the event horizon, more Hawking radiation is radiated away into cold empty space than the surrounding space radiates back into the black hole. The black hole literally evaporates away because of its temperature. As Hawking radiation is radiated away into cold empty space, the mass of the black hole decreases and the size of its event horizon decreases. Eventually the black hole will evaporate away into nothing. The puzzle was what happens to the information for all the stuff that originally fell into the black hole. Where does that information go as the black hole eventually evaporates away into nothing? This puzzle is the famous information loss paradox.

The problem with the information loss paradox is it assumes things actually carry information with them as they appear to fall into the black hole. Things are characterized by information and so it would seem that information accumulates inside the black hole as things appear to fall into the black hole. The assumption is that all of that information ends up inside the black hole.

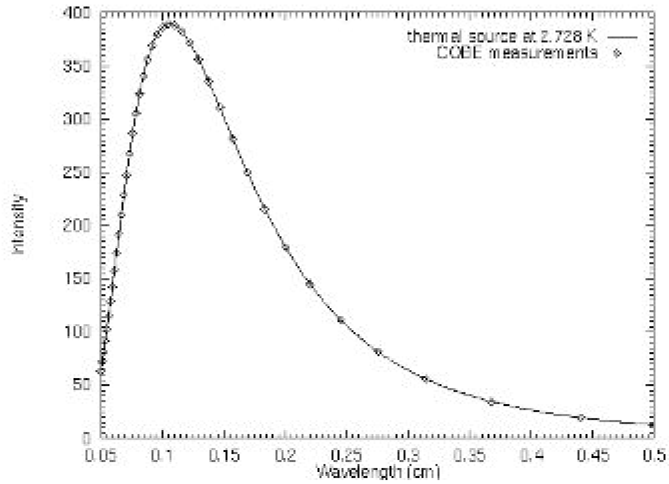
As is well known, appearances are deceiving. Things are characterized by information, but that information does not actually exist within things. The solution to the information loss paradox is the holographic principle. The information that characterizes the appearance of things does not actually exist inside of things. Instead, all the information is encoded on a holographic screen. The appearance of anything in three dimensional space is a holographic projection of the image of that thing from a holographic screen to the point of view of the observer of that image. The projected images of things correspond to the organization of forms of information on the holographic screen. When the observer observes the image of anything, there is only the appearance of that thing actually existing in three dimensional space in the sense of holographic projection. When something appears to fall into a black hole, that's only another appearance that results from holographic projection. When the black hole appears to radiate Hawking radiation and the black hole appears to evaporate away into nothing, that's only another appearance that results from holographic projection. No matter what appears to happen, the information for those happenings is always encoded on the holographic screen. There is no loss of information.

There is no loss of information when a black hole appears to evaporate away into nothing. All the information is encoded on a holographic screen. That holographic screen can be understood as an event horizon like a cosmic horizon that arises in an observer's accelerated reference frame and which defines everything in the observer's world. The observer's holographic screen encodes information for the black hole, which appears to form as things appear to fall into it or as it appears to evaporate away into nothing or disappear through the radiation of Hawking radiation. The observer's holographic screen also encodes information for the things that appeared to fall into the black hole and for the Hawking radiation that's radiated away from the black hole. All that really happens is the information for the form of things that appear to fall into the black hole becomes reorganized into information for the form of a black hole which eventually becomes reorganized into information for the form of Hawking radiation. Information is not created or lost, but only reorganized into a different form. There is a caveat however. It is possible that the observer's holographic screen can increase or decrease in surface area, thereby creating or destroying information for the observer's world. How this happens is still a mystery, but there is evidence from the big bang event that such creation or destruction of information is possible. In a very deep sense, creation of the observer's world in a big bang event must create information for that world. Creation or destruction of information must occur far away from thermal equilibrium, and is probably best described by some sort of a non-equilibrium process like a phase transition.

There is a deep connection between the singularity of a black hole and the singularity of the big bang event. In a strict mathematical sense, the singularity of the big bang is the beginning of time and singularity of a black hole is the end of time. A black hole is a hole in space-time geometry, and time ends at the singularity. In the same way, the big bang event is the creation of space-time geometry, and time begins at the singularity. The black hole has a boundary, which is its event horizon. The observable physical universe created in the big bang event also has a boundary, which is a cosmic horizon. Just as the event horizon of a black hole is characterized by thermal blackbody radiation, the big bang event is also characterized by blackbody radiation, which we measure as the cosmic microwave background radiation. In a deep sense, this thermal blackbody radiation of the physical universe is related to the temperature of the cosmic horizon. In some sense the cosmic microwave radiation is the Hawking radiation of the cosmic horizon, or at least it will become so when the physical universe eventually comes to thermal equilibrium.



Cosmic Microwave Background Radiation



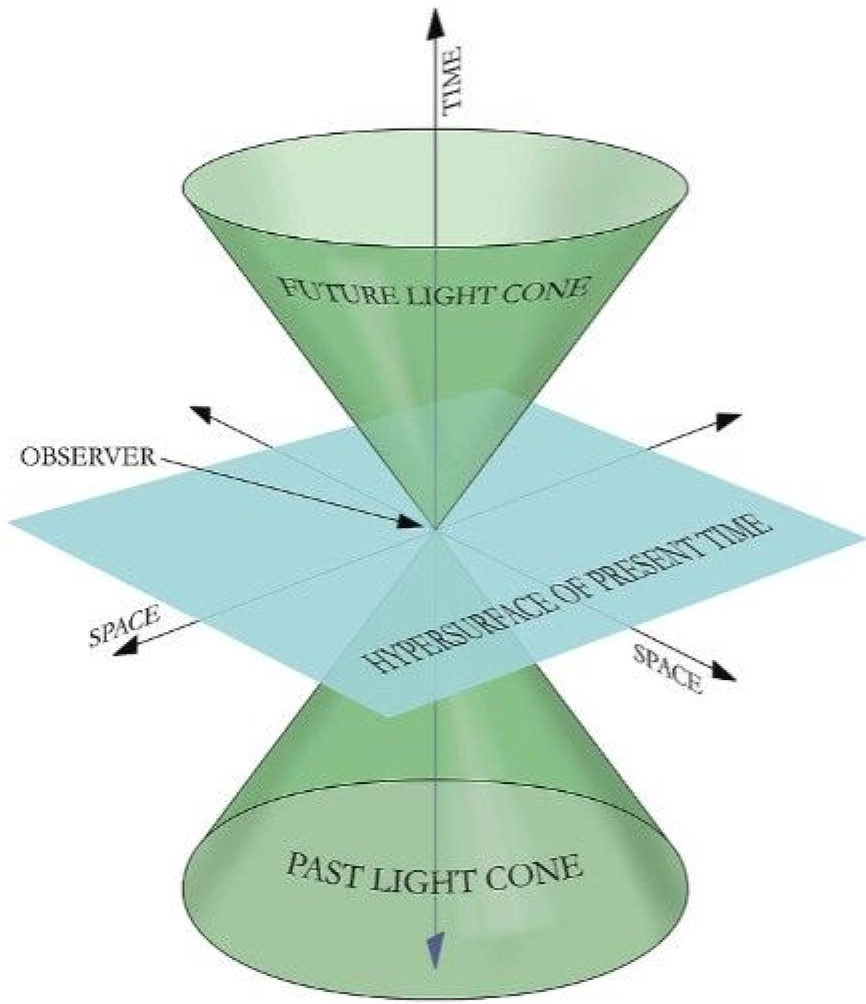
Blackbody Spectrum of CMB

We pick up the story again in the 1990's when Gerard 't Hooft and Leonard Susskind proposed the holographic principle as the solution to the information loss paradox. Shortly after they proposed the holographic principle, a remarkable discovery was made. This discovery was the AdS/CFT correspondence, which was a mathematically exact formulation of the holographic principle that at least in a particular situation made manifest the nature of the holographic screen.

Susskind and 't Hooft were able to solve the information loss paradox because they closely examined what it means to be an observer in an accelerated reference frame. They realized that an external observer of the black hole must be in an accelerated reference frame due to the force of gravity exerted by the black hole. This is like an observer in a rocket-ship that uses the force of its thrusters to hover above the event horizon of the black hole. If the thrusters of that rocket-ship are turned off, the rocket ship will fall into the black hole due to the unopposed force of gravity. In the first case, the observer is in an accelerated frame of reference and in the second case the observer is in a state of freefall, which is called a freely falling frame of reference.

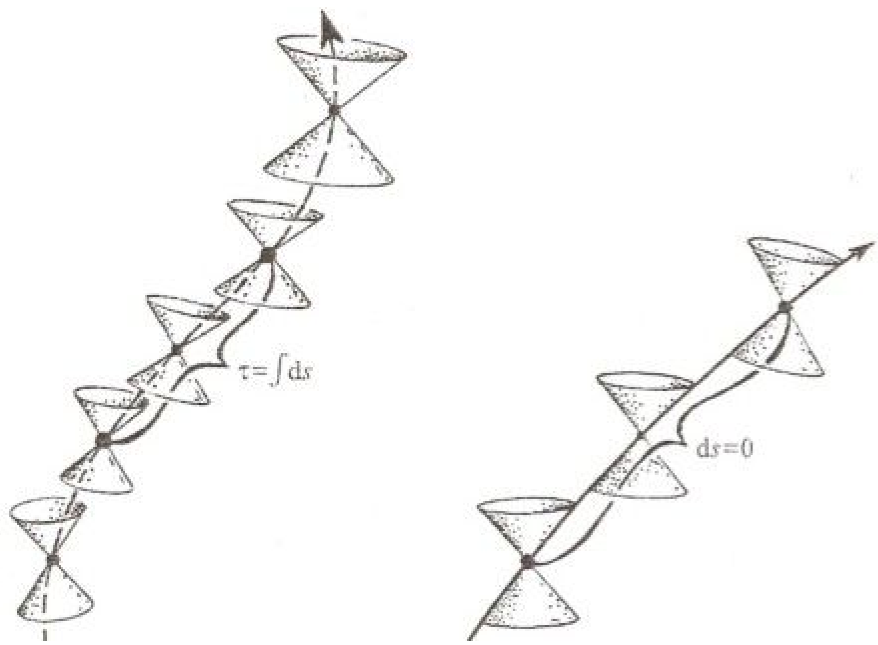
The external observer in the accelerated frame of reference observes the emission of Hawking radiation from the event horizon of the black hole. In the sense of Hawking's heuristic derivation, this corresponds to the separation of virtual particle-antiparticle pairs at the event horizon. The virtual antiparticle falls into the black hole while the virtual particle travels to the point of view of the external observer and is observed to be a real particle with the characteristics of thermal blackbody radiation. The big question is what does a freely falling observer that falls through the event horizon of the black hole observe? The answer is nothing. As far as the freely falling observer is concerned, there are no particles of Hawking radiation. There isn't even an event horizon. The event horizon is only an imaginary surface in space. For the accelerated external observer, the event horizon and the particles of Hawking radiation are real and appear to exist, but for the freely falling observer, those things don't even appear to exist. How is that possible?

To address the issue of how something can appear to exist for one observer but not for another observer, we need to discuss the nature of causality. In relativity theory, causality is a statement that past events influence future events. The way this is represented is in terms of the light cone. The light cone is defined in terms of an observer at the central point of view. The light cone is comprised of all past and future events that observer can observe. The observer's observations of events is always limited within the light cone of past and future events due to the limitation of the speed of light as a means of information transfer in three dimensional space. An event that falls outside the light cone is not observable to the observer at the central point of view since no light ray can ever connect that unobservable event to the observer at the central point of view.



Observer's Light Cone of Past and Future Events

From the point of view of any observer, causality is a statement that past events can influence future events only if the events fall within the observer's light cone of past and future events. An event is only observable to the observer if that event falls within the observer's light cone of past and future events. Only events that fall within the observer's light cone of past events can influence the observer's observations of events in its light cone of future events. Inherent in this description of causality is an assumption that the observer is actually observing something when the observer observes an event. In the sense of particle physics, that thing is a point particle that follows a world-line through the space-time geometry the observer observes. In relativity theory, the world-line the point particle follows through space-time is determined as a path that maximizes the proper-time interval, which is equivalent to minimizing the action.



Proper-time Intervals Determine Particle Paths through Space-time

Causality is a statement that particle paths observed in the past influence particle paths observed in the future. The events the observer observed in its past light cone influence events observed in its future light cone since particle paths are determined by maximizing the proper-time interval. Inherent in this statement of causality is an assumption that point particles actually exist. That assumption must be made for different observers in different frames of reference to observe the same particle paths. When past and future light cones of different observers in different frames of reference overlap, it is natural to assume observers will observe the same particle paths as long as those paths fall within the respective past and future light cones of the different observers.

The problem is a freely falling observer that falls across the event horizon of a black hole does not observe the same events as a stationary accelerated observer outside the event horizon. The stationary external observer outside the event horizon observes particles of Hawking radiation radiated away from the event horizon of the black hole and the freely falling observer doesn't. From the perspective of the external stationary observer, particles of Hawking radiation appear to exist, but from the perspective of the freely falling observer, particles of Hawking radiation do not exist. From the perspective of the external stationary observer, the event horizon of the black hole appears to have a temperature and to radiate away thermal particles of Hawking radiation, but the freely falling observer observes none of these things.

Causality is a statement that things actually exist. In relativity theory, causality is a statement that particles actually exist and all observers will measure the same particle paths with the same proper-time intervals. When observers don't even agree on the existence of particles, causality is violated. How can something appear to exist for one observer and not for another observer?

The answer is the holographic principle and the illusory nature of holographic projection from a holographic screen. There are many examples of weirdness in modern physics. For example, quantum uncertainty implies the end of determinism. Determinism is not a valid concept since we only talk about quantum probability. The observable world is inherently probabilistic and not deterministic. Another example is the end of simultaneity implied by relativity. The timing when events are observed to occur can appear different for different observers in different frames of reference. The time intervals separating events can appear different for different observers that observe those events in different reference frames.

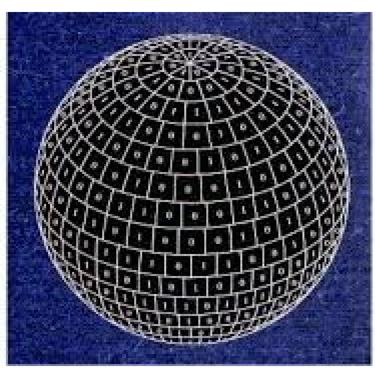
The end of determinism and the end of simultaneity are weird enough, but the end of causality implied by the holographic principle is the weirdest example of all. The holographic principle implies the end of causality precisely because what one observer observes may not be what another observer observes. Different observers in different frames of reference observe different things even when they're observing what they think are the same events. The events that appear to exist for one observer may not be the same events that appear to exist for the other observer.

Appearances are deceiving because of the way information is encoded for the appearance of things on a holographic screen. Things are characterized by information, but that information does not actually exist within things. The information that characterizes the appearance of things does not actually exist inside of things. Instead, all information is encoded on a holographic screen. The appearance of anything in three dimensional space is a holographic projection of the image of that thing from a holographic screen to the point of view of the observer of that image. The projected images of things correspond to the organization of forms of information on the holographic screen. When the observer observes the image of anything, there is only the appearance of that thing actually existing in three dimensional space in the sense of holographic projection. When something appears to fall into a black hole, that's only another appearance that results from holographic projection. When the black hole appears to radiate Hawking radiation, that's only another appearance that results from holographic projection. No matter what appears to happen, the information for those happenings is always encoded on the holographic screen.

The principle of equivalence tells us that every frame of reference has equal validity. There is no absolute of preferred frame of reference. What the external accelerated observer observes is just as valid as what the freely falling observer observes. The problem is the accelerated external observer observes a black hole event horizon that has a temperature and that radiates Hawking radiation, while the freely falling observer doesn't. The freely falling observer doesn't observe any of this stuff. As far as the freely falling observer is concerned, what the accelerated external observer observes doesn't even exist.

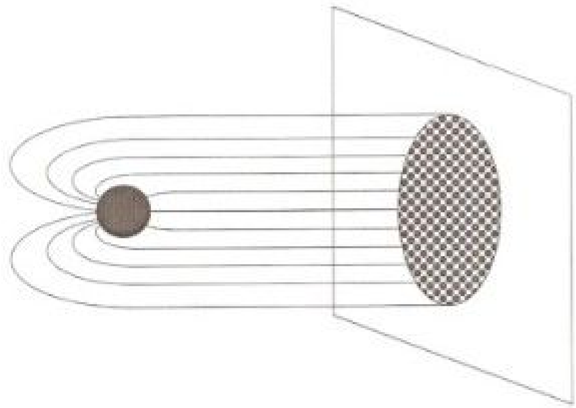
How can this state of affairs even be possible? How can two observes disagree about the nature of what exists? One possible answer is once the freely falling observer crosses the event horizon of the black hole, there is no possibility of communication between the freely falling observer inside the black hole and accelerated external observer outside the black since no signal that originates from inside the black hole can ever cross the event horizon. The two observers cannot communicate with each other about their differing observations of what exists, and so there is no possibility of disagreement. This question about what really exists becomes meaningless.

The other possible solution to this problem is the holographic principle. In the strict sense of ontology, nothing perceivable really exists. Everything perceivable is only a holographic illusion that results from holographic projection from a holographic screen to the point of view of an observer. The perceivable things are all forms of information that are encoded and organized on the holographic screen. The perception of anything is a projection from the holographic screen to the point of view of the observer. These perceptions are illusory in the same sense the projection of movie images from a movie screen to an observer out in the movie audience is illusory. The observer itself is not a perceivable thing, but can only be described in the sense of perceiving consciousness that arises at a point of view in relation to the screen. Consciousness itself is not a perceivable thing. Consciousness is what perceives things. We can't say what consciousness is in the framework of perceivable things because it isn't a perceivable thing.



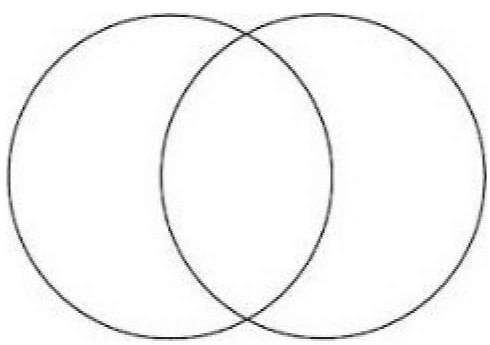
Horizon Information

The perceivable things are all holographic projections from the holographic screen, where all the information for those things is encoded. Different observers in different frames of reference can observe different things due to the nature of holographic projection. Different observers can disagree about the nature of what appears to exist since what appears to exist is only an illusory holographic projection. Every observer is observing the form of things as projected from its own holographic screen. Those perceptions don't necessarily need to agree since the way information is encoded and organized on different screens can be different for different observers.



Holographic Projection

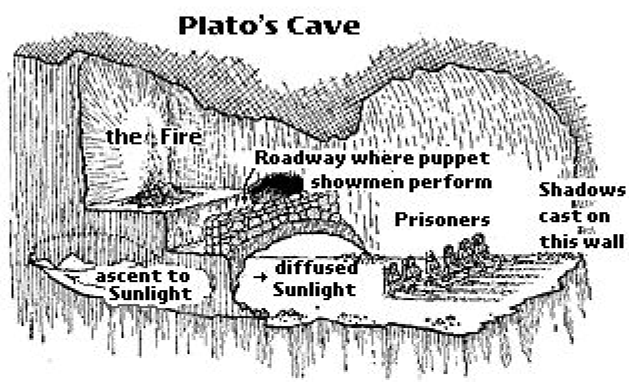
The fact that different observers can observe different things and can even disagree about what appears to exist is not surprising since that is a natural consequence of the holographic principle. What is surprising is that different observers can share a consensual reality and can agree about anything. Again, the holographic principle suggests a solution. Each observer observes events as projected from its own holographic screen, but in the sense of a Venn diagram those screens can overlap and share information, much like the kind of information sharing we see in a network of computer screens, like the internet. A consensual reality shared among different observers becomes possible with information sharing among overlapping holographic screens.



Overlapping Bounding Surfaces of Space

When Susskind and 't Hooft first proposed the holographic principle it was only a good idea in search of a more rigorous mathematical solution, which was soon discovered with the AdS/CFT correspondence. This rigorous mathematical solution about the nature of a holographic screen was possible because of the special properties of anti-de Sitter space, which is a geometric space that only has a single boundary, called an anti-de Sitter event horizon, and a single central point of singularity, which is the central point of view of an observer. The anti-de Sitter horizon is the observer's holographic screen. The special properties of anti-de Sitter space is what reduces the complexity of the problem and makes an exact mathematical solution possible. In anti-de Sitter space, we don't have to worry about differing observations of different observers because there aren't any. In anti-de Sitter space there is only a single observer and a single holographic screen.

In the Allegory of the Cave, Plato describes the perceivable world in terms of shadows projected on the wall of a cave. Plato describes a source of light that projects shadows on the wall of the cave and describes prisoners that perceive these projected shadows and identify themselves with the shadows. In this allegory, the cave is the observable universe and the wall of the cave is a cosmic horizon that defines the boundary of the observable universe relative to an observer at the central point of view of the observable universe. As a cosmic horizon, the wall of the cave limits the observer's observations of things in space. In the sense of the holographic principle, the wall of the cave acts as a holographic screen that projects the perceivable images of all things in the observer's world to the observer's central point of view. These projected images are forms of information, which Plato refers to as shadows. The projecting light is the light of consciousness and the observer is the perceiving consciousness. Plato refers to the observer as a prisoner since the observer identifies itself with the projected image of a shadow it perceives.



In what sense are images projected from a holographic screen shadows? The formulation of the holographic principle in the AdS/CFT correspondence gives an answer. If all perceivable objects are really defined on a two dimensional holographic screen in terms of where all the fundamental bits of information are encoded, then where does the third dimension come from? The answer is given by the correspondence between conformal field theory (CFT) and gravity in anti-de Sitter space (AdS). The perception of gravity in a bounded region of anti-de Sitter space is equivalent to a conformal field theory encoded on the bounding surface of that space. The perception of a third dimension arises from the Weyl symmetry of the conformal field theory. Conformal invariance is not a symmetry of space-time geometry the way Lorentz invariance is a symmetry of flat Minkowski space, but is a symmetry of the space-time metric that measures the curvature of space-time geometry. Conformal invariance allows anti-de Sitter space to become a holographic space in the sense that whatever appears to happen in that space is a holographic projection from the bounding surface, where all the information for those happenings is encoded, to the central point of view of the observer that perceives those happenings.

Weyl symmetry of the space-time metric is what gives rise to the perception of a third dimension. Conformal invariance is inherently a symmetry of the changing size of objects, which are forms of information. Conformal symmetry is also inherent in the geometric structure of fractals, which appear the same or self-similar no matter what distance scale they're looked at. A small part of a fractal looks like the same geometric structure as a large part of the fractal. In the AdS/CFT correspondence, all objects in space exhibit conformal symmetry. As objects in space appear to move toward or away from the point of view of an observer and appear to grow larger or smaller in size, the way bits of information are encoded for the objects on the bounding surface of that space also grow larger or smaller in size in exact proportions of three dimensional perception. It is as though a light is projecting a shadow of the object onto a screen, and that shadow is growing larger or smaller in size as the object moves toward or away from the observer. The holographic principle is telling us that the shadow is the nature of the perception of objects. Objects don't really exist in three dimensional space except as holographic projections. Their shadows only exist in terms of the information encoded on a two dimensional bounding surface. This way of describing the appearance of objects in space as shadows projected on a wall is eerily similar to how Plato describes objects in the Allegory of the Cave.

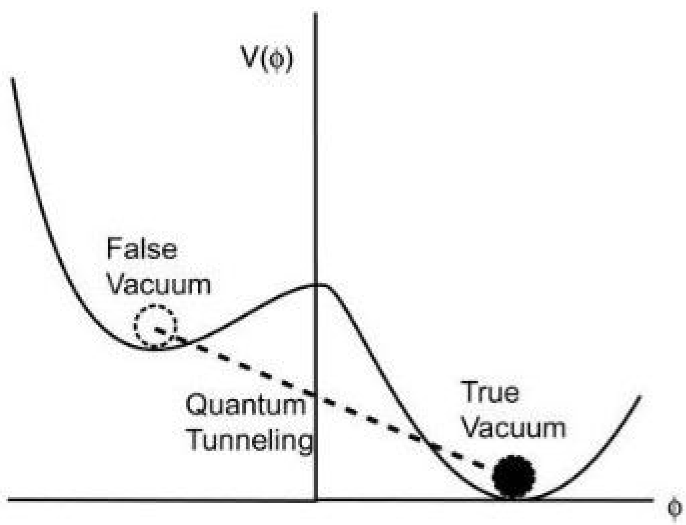
Anti-de Sitter space arises as a solution to Einstein's field equations for the space-time metric with a negative cosmological constant. In Einstein's theory of gravity, gravity is a locally attractive force that causes the local contraction of space. A negative cosmological constant gives rise to a globally attractive force that causes the global contraction of space. In a heuristic sense, anti-de Sitter space arises from the accelerated contraction of space.

Anti-de Sitter space is weird. In a strict mathematical sense, the distance from the central point of singularity to the anti-de Sitter event horizon is infinite, but due to the accelerated contraction of anti-de Sitter space, it takes light a finite amount of time to travel this infinite distance. In a strict mathematical sense, the anti-de Sitter event horizon is a boundary at infinity, but in the sense of the holographic principle, it encodes a finite amount of information for everything that can appear to happen within that bounded region of space. In a very deep sense, anti-de Sitter space is unphysical since it is mixing up the finite with the infinite. This is not a physical space.

The problem with anti-de Sitter space is the negative value of the cosmological constant. The cosmological constant is understood as a vacuum energy, which is the energy of empty space. For a physical space, the lowest possible value of the vacuum energy is zero. A negative vacuum energy does not correspond to a physical space. It is possible in physics to have false vacuums that correspond to metastable states in which the vacuum energy is non-zero, but for these metastable states to be physical, the vacuum energy of these false vacuums must be positive.

The observational evidence is that the observable physical universe does indeed exist in a metastable state or in a false vacuum state with a positive vacuum energy. This positive vacuum energy is the measured cosmological constant, which in Planck units has a numerical value of about 10^{-123}. This value is determined from observations of the rate with which distant galaxies are accelerating away from us, which corresponds to the accelerated expansion of space. There is also evidence from observations of the cosmic microwave radiation left over from the big bang event that early in the history of the universe, the cosmological constant had a value of about 1.

There must be some mechanism that allows the false vacuum state of the observable physical universe to transition from a higher value of vacuum energy to a lower value of vacuum energy. This would be a transition from a less stable metastable state to a more stable metastable state. Such a transition must have occurred early in the history of the universe to reduce the initial value of the cosmological constant to its current measured value of about 10^{-123}. There may have been more than one transition, and even more transitions may be possible since the most stable state is the true vacuum with a zero value of vacuum energy. The transition mechanism is unknown, but most likely it is a non-equilibrium process like a phase transition.

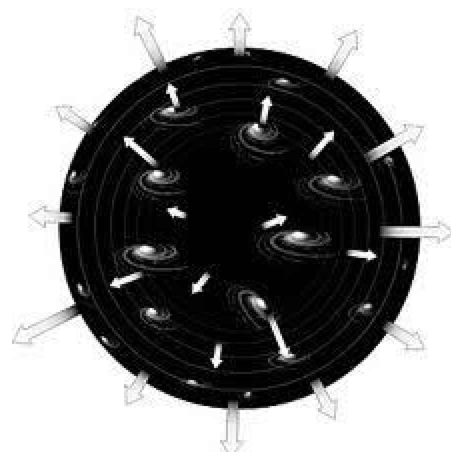


Metastable State

A non-zero positive value for the cosmological constant corresponds to a metastable state with a positive vacuum energy that through some unknown mechanism can decay into a more stable state with a lower energy. Since this vacuum energy is the energy of empty space, the lowest possible value that can correspond to a physical space is zero. A non-zero positive vacuum energy is a false vacuum, while a zero vacuum energy is the true vacuum. A negative vacuum energy is not physical, and cannot correspond to the physical universe.

The problem with the AdS/CFT correspondence is the kind of cosmological space we find ourselves within inside the physical universe is not anti-de Sitter space but de Sitter space. In relativity theory, anti-de Sitter space arises with a negative cosmological constant, which gives rise to a globally attractive force that corresponds to the accelerated contraction of space, while de Sitter space arises with a positive cosmological constant, which gives rise to a globally repulsive force that corresponds to the accelerated expansion of space. For various reasons, a positive cosmological constant is now called dark energy, and de Sitter space is understood as the accelerated expansion of space that arises as dark energy is expended.

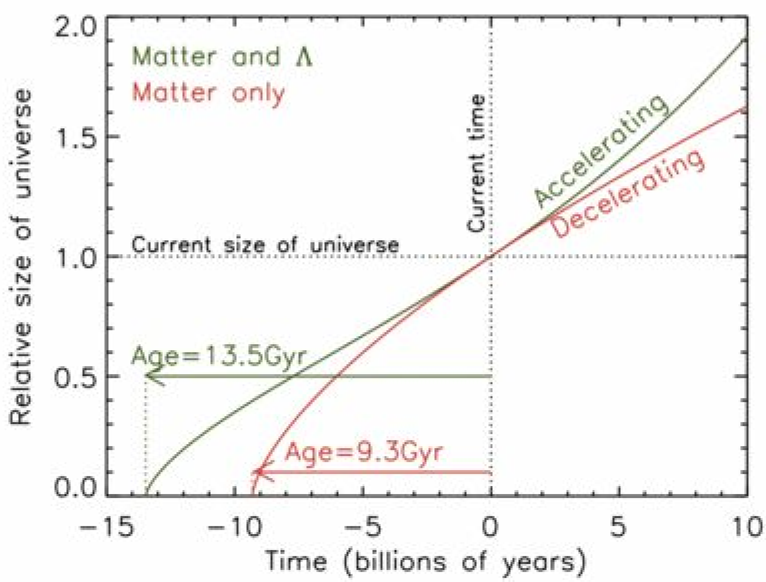
As dark energy is expended, the observer at the central point of view enters into an accelerated frame of reference and space appears to expand away from the observer at an accelerated rate, faster the farther out the observer looks into space. At some point in space, space appears to expand away from the observer at the speed of light, and nothing is observable beyond that point, which defines the surface of the observer's cosmic de Sitter horizon. The accelerated observer is surrounded by a cosmic horizon, which is a bounding surface of space that limits the observer's observations of things within that bounded region of space. Nothing is observable beyond the observer's cosmic horizon due to the accelerated expansion of space, which in relativity theory is understood as an exponentially expanding universe.



Accelerated Expansion of Space in an Exponentially Expanding Universe

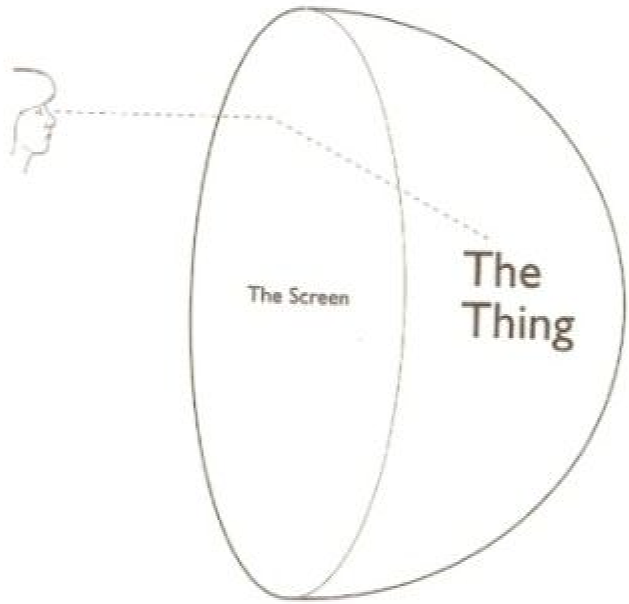
The space-time metric for de Sitter space is a solution of Einstein's field equations with an extra cosmological term in the equations of the form Λg_{ab} where Λ is the cosmological constant. The space-time metric takes the form $(\Delta\tau)^2=(\Delta t)^2-\exp(\alpha t)[(\Delta x)^2+(\Delta y)^2+(\Delta z)^2]/c^2$ where the parameter $\alpha^2=c^2\Lambda/\ell^2$ is expressed in Planck units. The exponential factor is what gives rise to the exponential expansion of space. The de Sitter metric can also be expressed in terms of spherical coordinates and a radius r as $(\Delta\tau)^2=-(1-r^2/R^2)(\Delta t)^2+(1-r^2/R^2)^{-1}(\Delta r)^2/c^2$. The reversal of the minus sign in the metric indicates we're observing things from inside of de Sitter space. The curvature radius $R^2=3\ell^2/\Lambda$ specifies the distance from the observer's central point of view to the observer's cosmic horizon. In terms of the holographic principle, the cosmic horizon has an entropy and encodes n bits of information as $n=A/4\ell^2=4\pi R^2/4\ell^2=3\pi/\Lambda$, and has a temperature given by $kT=\hbar c/2\pi R$. In this solution for the de Sitter metric, the observer's central point of view is located at the origin of de Sitter space r=0, which is central to the cosmic horizon.

Why can't the cosmological constant be derived from the laws of physics, like the idea that dark energy is a vacuum energy that arises from the quantum fluctuations of some quantum field? The answer is the cosmological constant is the boundary condition that sets the laws of physics. The laws of physics can only thermodynamically emerge once the boundary condition is set. This boundary condition has to be set a priori. The observational evidence is that we do indeed live in an exponentially expanding universe characterized by the accelerated expansion of space and a de Sitter cosmic horizon. The measured value for Λ is about 10^{-123} based on the rate that distant galaxies are observed to accelerate away from us. This gives a distance to the cosmic horizon of about 60 billion light years and an entropy of about 10^{124} bits of information.



Accelerating Universe

The holographic principle tells us the observer’s cosmic horizon acts as a holographic screen that encodes all the bits of information for all the observable things the observer can observe in that bounded region of space. The observation of anything within that bounded region of space is a holographic projection of a form of information projected like an image from the screen to the observer’s central point of view, which is at the center of that bounded region of space.



The Observer's Holographic Screen

The AdS/CFT correspondence explicitly demonstrates the holographic principle in anti-de Sitter space, but for various technical reasons cannot be generalized to de Sitter space. The problem is anti-de Sitter space only has a single observer at the central point of singularity and a single anti-de Sitter horizon at infinity, while de Sitter space allows for multiple observers and multiple finite horizons. Each observer in de Sitter space is surrounded by its own cosmic horizon, but there can by many observers with overlapping cosmic horizons. There is no single boundary at infinity. It is possible to extend the holographic principle into de Sitter space. The AdS/CFT correspondence is a special case of non-commutative geometry, and generic non-commutative geometry can be applied to de Sitter space, which is to say non-commutative geometry can be applied to the kind of space we find ourselves within inside the physical universe. Even fractal geometries can be understood as special cases of non-commutative geometry. Through the magic of non-commutative geometry, the holographic principle can be extended into de Sitter space.

It may seem like an arcane topic of discussion, but non-commutative geometry is the natural way to understand how space-time geometry is quantized. The natural kind of geometry for which non-commutative geometry can be applied is a two dimensional bounding surface of space, like an event horizon. Instead of localizing an infinite number of infinitesimal points on the surface of the horizon, with non-commutative geometry a finite number of quantized position coordinates are defined on the surface. In effect, each quantized position coordinate defined on the surface is smeared out into an area element, like a pixel on a screen, with a well-defined mathematical procedure for defining quantized position coordinates in terms of non-commuting variables.

In quantum gravity, the pixel size is about a Planck area, and the total number, n, of quantized position coordinates defined on the surface is given in terms of the surface area, A, as $n=A/4\ell^2$. Non-commutative geometry not only gives a mathematical procedure for how quantized position coordinates are defined on the surface in terms of non-commuting variables, but also explains how each quantized position coordinate acts like a pixel that encodes a bit of information in a binary code of 1's and 0's. The bounding surface typically encodes n bits of information as the n eigenvalues of an SU(n) matrix. Just as the two eigenvalues of an SU(2) matrix can explain how a spin variable is quantized into spin up and spin down states, like a computer switch that is either on or off and encodes information in a binary code, the n eigenvalues of an SU(n) matrix can explain how n non-commuting variables defined on the surface encode n bits of information.

Since all the information for a holographic world arises as the eigenvalues of an SU(n) matrix, all the bits of information are naturally entangled in the sense of quantum entanglement. All the paradoxes of quantum entanglement that Einstein referred to as spooky action at a distance have a natural explanation in terms of holographic projection. Entangled objects in a holographic world can appear to separate in distance, but the entangled bits of information that define those objects as encoded on a holographic screen do not separate, and so there is no paradox. The appearance of the separation of objects is an illusion that results from holographic projection.

Non-commutative geometry gives us a natural operational explanation for how the holographic principle comes into effect. Whenever non-commutative geometry is applied to a bounding surface of space as a way to define n quantized position coordinates on the bounding surface, the holographic principle is automatically in effect and the bounding surface encodes n bits of information in a binary code of 1's and 0's, typically as the n eigenvalues of an SU(n) matrix. Each non-commuting variable defined on the bounding surface acts like a pixel that encodes a bit of information. The bounding surface of space naturally arises as an event horizon whenever an observer enters into an accelerated frame of reference, such as a cosmic de Sitter horizon that arises whenever dark energy is expended and space appears to expand at an accelerated rate away from the central point of view of the observer at the central point of singularity.

This operational explanation explains the nature of everything the observer can observe within the bounded space, which in effect defines the observer's world. The nature of the observation of anything within the bounded space, which is the nature of everything the observer can observe within the bounded space that arises in its accelerated frame of reference, is a holographic projection from the observer's holographic screen, which is a bounding surface of space, to its central point of view, which is the central point of singularity of that bounded space.

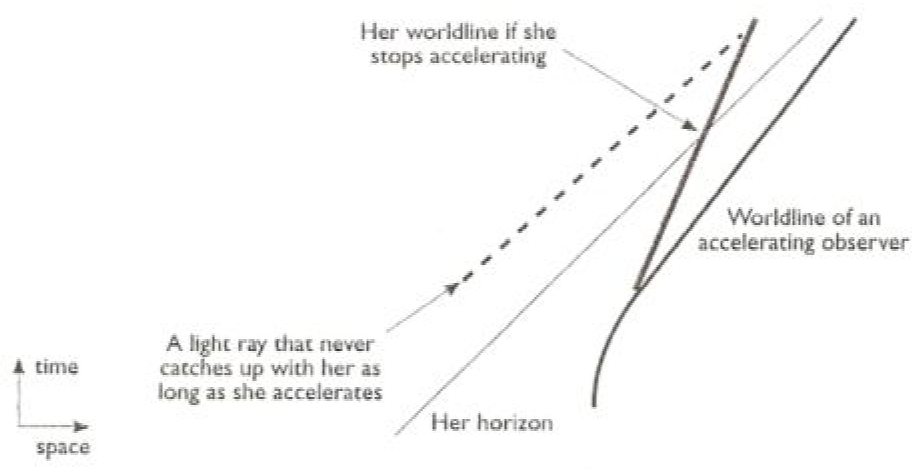
The holographic principle says that all the bits of information that describe the configuration states for everything perceivable in the world are encoded on a bounding surface of space that acts as a holographic screen. These bits of information are the dynamical degrees of freedom that are quantized in quantum theory, which in thermodynamics are called entropy. These bits of information are naturally entangled due to the way they're encoded on a holographic screen. The easiest way to understand the nature of this holographic encoding of bits of information on a bounding surface of space is with non-commutative geometry.

Ted Jacobson has shown the dynamical nature of space-time geometry in any bounded region of space, which is the nature of gravity, is a thermodynamic consequence of the holographic way bits of information are encoded on the bounding surface of that space. The reason is quite simple. As energy flows across a bounding surface of space, the second law $\Delta Q=T\Delta S$ tells us the entropy of that bounded region must change, but the holographic principle then tells us the bounding surface must change, which is reflected in a change in the geometry of that bounded region of space as specified by Einstein's field equations for the space-time metric. The law of gravity as formulated with Einstein's field equations are not really laws at all, but are more like thermodynamic equations of state that arise as thermal averages when things are near thermal equilibrium. This is analogous to the way wave equations for sound waves arise from atomic theory as thermodynamic equations of state. Einstein's field equations for gravity are no more fundamental than wave equations for sound waves. The microscopic formulation of bits of information encoded on a bounding surface of space is more fundamental than Einstein's field equations just as atomic theory is more fundamental than wave equations for sound waves. The scientific term for how field equations arise from information is thermodynamic emergence.

Understanding that everything observed in any bounded region of space is a thermodynamic consequence of the holographic way bits of information are encoded on the bounding surface of that space tells us that everything spontaneously emerges in the flow of energy, which is fundamentally the flow of heat. The bounding surface can only arise in the flow of energy, which in relativity theory is understood as an event horizon that arises in the observer's accelerated frame of reference, which always implies the expenditure of energy.

Due to the limitation of the speed of light as a means of information transfer in three dimensional space, every observer in an accelerated frame of reference is surrounded by an event horizon that limits the observer's observations of things in space. The event horizon is as far out into space as the observer can see things in space. The event horizon is a bounding surface of space, which can be understood as a holographic screen, like a computer screen.

Every accelerating observer has its own event horizon, which limits its observations of things in space. The observer's event horizon is observer-dependent in the sense it can only arise in the observer's accelerated frame of reference. For example, an observer in de Sitter space has a cosmic de Sitter horizon that arises due to the accelerated expansion of space. Even an observer in empty space that undergoes a constant acceleration has an event horizon called a Rindler horizon that limits the observer's observations of things in space.



Accelerating Observer's Horizon

The observer's event horizon is a bounding surface of space that limits the observer's observations of things in space. When non-commutative geometry is applied to that bounding surface as a way to specify a finite number of quantized position coordinates on the screen, each position coordinate is smeared out into an area element like a pixel on a screen that encodes a bit of information in a binary code of 1's and 0's. The holographic principle tells us everything the observer observes within that bounded region of space is like a holographic projection of images from the screen to the point of view of the observer. The observable images of things are forms of information projected to the observer from the screen and animated in the flow of energy, just like the images of a movie. The bits of information encoded on the screen are the dynamical degrees of freedom for everything observed within the bounded space.

These degrees of freedom are the dynamical variables that are quantized in quantum theory, which in thermodynamics are called entropy. Unlike the idea of particle physics inherent in classical physics and ordinary quantum theory, the dynamical variables are no longer particle coordinates, but are bits of information encoded on the screen. Only geometric concepts are needed to understand the nature of information, but another concept is needed to understand the nature of energy. That idea is temperature and thermal energy, which is the idea of heat. At thermal equilibrium, each degree of freedom in any bounded region of space has an average amount of thermal energy that defines temperature as $E=kT$. The total amount of thermal energy in the bounded region of space, which is called the heat content Q, is the total number of degrees of freedom n multiplied by the amount of thermal energy per degree of freedom, which is kT. This gives the second law of thermodynamics as $Q=nkT$, or $\Delta Q=T\Delta S$, where entropy $S=kn$.

Using $\Delta Q=T\Delta S$ along with the holographic principle, which says the total number of degrees of freedom is defined in terms of the surface area A of the bounding surface as $n=A/4\ell^2$ where the

Planck area $\ell^2=\hbar G/c^3$, and the Unruh formula for the temperature of an event horizon that arises in an observer's accelerated frame of reference $kT=\hbar a/2\pi c$ where a is the observer's acceleration, turns these concepts of thermal energy in purely geometric concepts.

In quantum theory, the Unruh effect is understood as a kind of Hawking radiation that results from an accelerating observer observing the separation of virtual particle-antiparticle pairs at the observer's event horizon, which turns separated virtual particles into a kind of thermal radiation. The classic way to understand the Unruh effect is for a Rindler horizon that arises for an observer undergoing constant acceleration. The Unruh effect in effect quantizes the thermal energy of each degree of freedom as $E=kT=\hbar\omega$, where the natural frequency of oscillation is given in terms of the observer's acceleration as $\omega=a/2\pi c$. It is easy to show that for the event horizon of a black hole of radius R this frequency of oscillation implies a wavelength λ as $\omega=2\pi c/\lambda$, where the wavelength is approximately the maximal circumference $2\pi R$ of the event horizon, which is characteristic of Hawking radiation. This tells us the temperature of the event horizon is inversely proportional to its radius. The other way to look at this result is the Unruh effect implies gravitational acceleration and potential energy if we understand that the wavelength of thermal radiation from an event horizon is quantized in terms of its circumference.

Hawking found the temperature of a black hole's event horizon is given in terms of its radius as $kT=\hbar c/4\pi R$, where the radius of the event horizon is given in terms of the mass of the black hole as $R=2GM/c^2$. If the acceleration due to gravity at the event horizon is defined as $g=GM/R^2$, this temperature can be written in the form of an Unruh temperature $kT=\hbar g/2\pi c$. The thermodynamic description of the black hole is completed by the expression for entropy $S=kn=kA/4\ell^2$ and the second law of thermodynamics $\Delta Q=T\Delta S$. This formulation generalizes to any event horizon that arises in an observer's accelerated reference frame, like a cosmic horizon that arises in de Sitter space or a Rindler horizon that arises in empty space with constant acceleration.

The amazing thing is the second law of thermodynamics interpreted in terms of the holographic principle and the Unruh temperature implies Einstein's field equations for the space-time metric, which is the law of gravity for everything that appears to happen in the bounded region of space. The law of gravity is understood to be a purely geometric result of the way bits of information are encoded on the bounding surface of that space and the temperature of the bounding surface. Einstein's field equations for gravity are not really fundamental, since they only arise as thermal averages valid near thermal equilibrium, like thermodynamic equations of state. Einstein's field equations for the space-time metric describe the dynamical space-time geometry of the bounded space, but everything observed within that bounded region of space is a holographic projection from the bounding surface of that space to the central point of view of the observer.

The usual unification mechanisms of modern physics like super-symmetry and the Kaluza-Klein mechanism then give a natural explanation for how all other fundamental forces and all fundamental particles arise from that dynamical space-time geometry as extra components of the space-time metric. Since the dynamical space-time geometry of the bounded space is derivative of the holographic principle, none of the so-called fundamental forces or particles are really fundamental. Everything observed in that bounded space arises through holographic projection.

There is a natural way to understand how holographic projection from a bounding surface of space gives rise to the perception of particles and fields in the bounded region of that space. The nature of holographic projection as the natural mechanism of observation is based on symmetry and symmetry breaking, where symmetry is inherent in a projected space-time geometry.

With ordinary space-time geometry, we have three independent directions of motion through space and one direction of motion through time. This can be represented as an x-y-z-t four dimensional geometry. Since time seems to be special, we call it 3+1 dimensional. This geometry has symmetries of translation through space and rotation of space, but when we add time in the geometric sense of 3+1 dimensional space-time, we get the full Lorentzian symmetries of space-time geometry. The conservation of energy and angular momentum come out of this geometric symmetry as a result of translational and rotational symmetries of space-time. Remarkably, the concept of particle spin also comes out of this Lorentzian symmetry as a symmetry of space-time rotation. The only difference between ordinary angular momentum and spin angular momentum is in terms of whether we rotate the x-y plane or the x-t plane.

Since all point particles carry spin and spin is a direct consequence of the Lorentzian symmetry of space-time rotations in the sense of a conservation law, we say all particles are representations of the Lorentz symmetry group. We can then classify particles in terms of their spins. The Higgs boson is a spin 0 particle; the electron, quarks and neutrinos are spin ½ particles; the photon, gluons and W and Z bosons are spin 1 particles, and the graviton is a spin 2 particle. This list of particles is all that ordinary quantum theory allows.

Super-symmetry is another weird aspect of modern physics. Super-symmetry adds another dimension to space-time, but this is a purely quantum dimension. The ordinary dimensions are represented by commuting numbers on a number line. Commuting numbers have the property that xy=yx, and so the order of multiplication doesn't matter. The new quantum dimension is described by anti-commuting numbers that have the property that xy=−yx, and so the order of multiplication does matter. With anti-commuting numbers, it's as though first stepping forward and then stepping to the right ends up in a different place than first stepping right and then stepping forward. The order with which the steps are taken matters to the end result. That's not the only weird thing about anti-commuting numbers. Since xy+yx=0, first taking the steps in one order and then taking the steps in the opposite order ends up nowhere since the end result is zero.

The strange thing about super-symmetry is it unifies the integer spin bosons with the half spin fermions. Every integer spin boson has a half spin fermion partner. There's something even weirder about super-symmetry that has to do with unifying all the fundamental forces. When the Kaluza-Klein mechanism of extra compactified dimensions of space is applied to Einstein's field equations for the space-time metric, the quantum fields of all the boson force particles naturally arise as extra components of the space-time metric. For example, with an extra compactified fifth dimension, Maxwell's field equations for electromagnetism naturally arise from Einstein's field equations in terms of the fifth component of the space-time metric. The equations for the strong and weak nuclear forces also arise in a similar way as long as there are a total of six extra compactified dimensions of space. When super-symmetry is also applied to Einstein's field equations, something even weirder happens. Not only do Maxwell's field equations pop out of Einstein's field equations, but Dirac's field equation for the electron also pops out. Not only do the field equations for the strong and weak nuclear forces pop out of Einstein's field equations, but the field equations for the quarks and neutrinos also pop out.

The Kaluza-Klein mechanism was discovered very shortly after Einstein discovered the field equations for the space-time metric. It's weird that both the Kaluza-Klein mechanism and the Schwarzschild solution for black holes have been around for over a hundred years and we're still trying to come to terms with what these discoveries really mean. The Kaluza-Klein mechanism was the first unification mechanism discovered. It's conventional to imagine a compactified dimension as a spatial dimension curled up into a small circle, but that's probably a mistaken concept. More likely, a compactified dimension is a non-commuting variable as represented by non-commutative geometry. Just like our ordinary experience of three dimensional space is an illusion that results from holographic projection, the concept of extra compactified dimensions of space is also probably another illusion that arises from non-commutative geometry.

In any case, when the usual unification mechanisms of super-symmetry and the Kaluza-Klein mechanism of extra compactified dimensions of space are applied to Einstein's field equations for the space-time metric, all the usual quantum fields of the standard model of particle physics naturally arise as extra components of the space-time metric. A quantum field is understood as an extra component of the space-time metric, which is a way of unifying all fundamental forces and particles into a unified theory of quantum gravity. We understand that a particle is a wave-packet of field energy and momentum directed through the usual extended dimensions of space.

The wave-particle duality of quantum theory is inherent in this quantum field theory description of particles, since a wave-packet sometimes acts like a particle when it's measured to be localized at some point in space-time, and sometimes acts like a wave when it's not well localized. The wave-packet of a quantum field is only a probability amplitude that specifies the probability with which the particle can be measured at some point in space-time. When the wave-packet is not well localized, multiple measurements result in wave-like behavior rather than particle behavior.

This wave-packet description of particles in quantum field theory has another weird aspect. Field energy and momentum can be quantized not only in the extended dimensions of space, but also in the extra compactified dimensions of space. The result of this quantization of momentum in a compactified dimension of space is what we call a charge, like the charge of the electron. Particles carry charges, like electric and nuclear charges, because momentum is quantized in the compactified dimensions that represent the behavior of the particle. Just as the spin angular momentum carried by particles is a result of the rotational symmetry of space-time, the charges carried by particles are a result of conservation laws that arise from the symmetry of space-time geometry when that symmetry is extended to include extra compactified dimensions.

Quantum field theory explains a lot about the nature of particles just as relativity theory explains a lot about the nature of space-time geometry. These theories are unified into a theory of quantum gravity with the usual unification mechanisms. The problem is neither the gravitational field nor any of the quantum fields are really fundamental as they all emerge through geometric mechanisms, starting with the holographic principle. A theory of quantum gravity is only a holographic description of what appears to happen in a bounded region of space. More fundamental than that description is the way bits of information are encoded on the bounding surface of that space and the flow of energy within which everything spontaneously emerges.

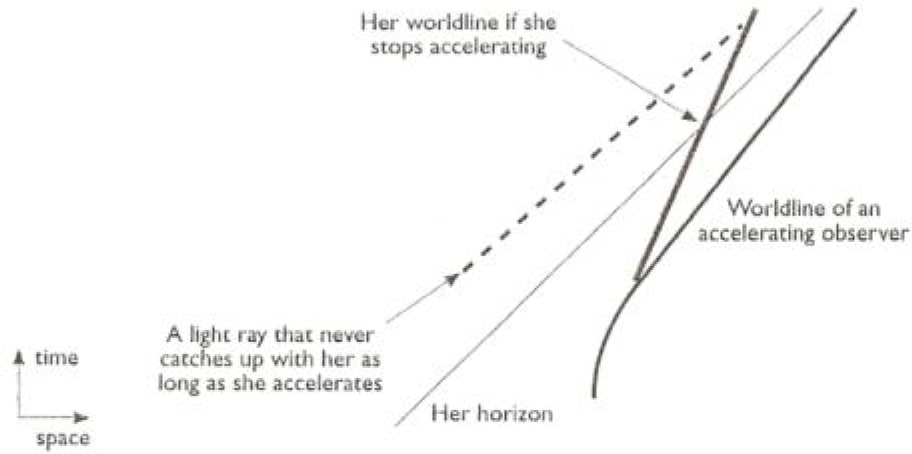
The concept of mass and energy as related by $E=mc^2$ is based on circular reasoning. We first have to assume the constancy of the speed of light for all observers, which is verified by experimental observation. The constancy of the speed of light implies time dilation, which is the effect of time appearing to run slower in different reference frames when two observers move with constant velocity v relative to each other. If two observers carry identical clocks, from the perspective of a stationary observer at rest relative to itself, a clock carried by a moving observer appears to run slower compared to the stationary observer's own clock. If Δt is a tick of the first observer's own clock while $\Delta t'$ is a tick of the second observer's clock as observed by the first observer, the equation for time dilation is $(\Delta t')^2=(\Delta t)^2/(1-v^2/c^2)$. The first observer observes the second observer's clock to run more slowly even when the two clocks are identical.

The second assumption has to do with how energy is defined. This is where circular reasoning comes into play. We need to define energy, and that requires some assumptions. In Newton's law of motion, $F=ma$, applying a force on a mass results in an acceleration of the mass. The natural definition of energy is the amount of work done ΔE as a force is applied through a distance Δx. This is written as $\Delta E=F\Delta x$. We write the force in terms of the momentum $p=mv$ as $F=\Delta p/\Delta t$, where the mass moves with a velocity $v=\Delta x/\Delta t$ and x is the position of the mass. If this was classical physics and the mass accelerates from a zero velocity to a velocity v as a force F is applied over the distance x, these relations would give the kinetic energy as $E=\frac{1}{2}mv^2$.

The circular reasoning comes into play since we are using the concept of force both to define energy as $\Delta E=F\Delta x$ and to define mass as $F=m\Delta v/\Delta t$. In relativity theory there really is no such thing as a force, only the relative accelerations that different observers experience relative to each other in different accelerated frames of reference. In reality, the only thing that really exists are observers in different accelerated frames of reference. That is exactly what the principle of equivalence says. We're only using the concept of force to relate mass and energy. Once a force is understood as an observer in an accelerated frame of reference, the concepts of energy and mass are essentially the same concept. Energy and mass are things that arise with accelerations. If we take this idea to its logical conclusion, we are then driven to the holographic principle.

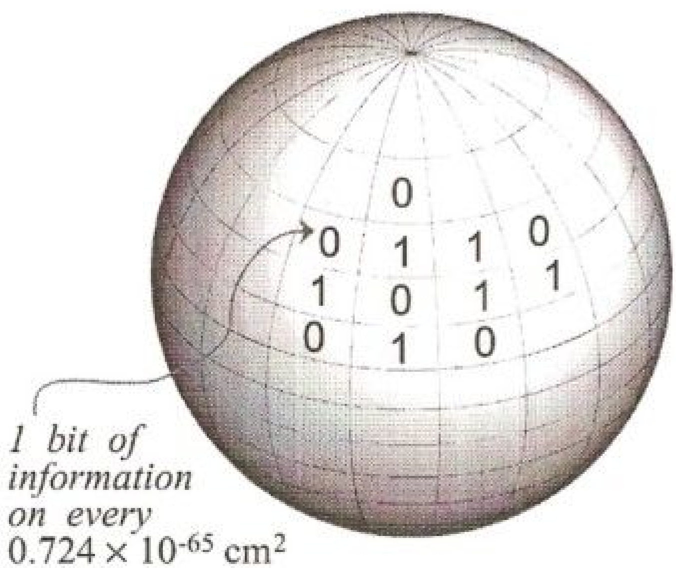
If we put these ideas of time dilation and this circular way of defining energy and mass in terms of a force as $\Delta E=F\Delta x$ and $F=m\Delta v/\Delta t$ together, it's possible to show that the general relation between energy and momentum, once all the forces are turned off, satisfies $E^2=p^2c^2+m^2c^4$. The relativistic momentum is given by $p=\gamma mv$ where $\gamma^2=1/(1-v^2/c^2)$ arises from time dilation. All of these quantities are defined as observed by a stationary observer when the mass moves with a velocity v. For v much less than c, this equation gives $E=mc^2+\frac{1}{2}mv^2+$... This is where the rest mass energy comes from, but all we're really doing is breaking up the total energy into kinetic energy and rest mass energy, which can be thought of as a kind of potential energy. There is no concept of force here, only different observers in different accelerated frames of reference.

The circular reasoning underlying $E=mc^2$ can also be understood in terms of the holographic principle. When we talk about a mass m that moves through space over time with a velocity v, we are really describing a projection of holographic images from a holographic screen to an observer's point of view. The holographic screen is an event horizon that arises in the observer's accelerated frame of reference and limits the observer's observations of things in space due to the limitation of the speed of light as a means of information transfer in three dimensional space. The observer's event horizon is as far out in space as the observer can see things in space. Every observer in an accelerated frame of reference has such an event horizon, which is called a Rindler horizon that arises because the observer is following an accelerating world-line. The observer's horizon in effect defines the observer's world in terms of whatever images of things can be projected from its holographic screen to its point of view.

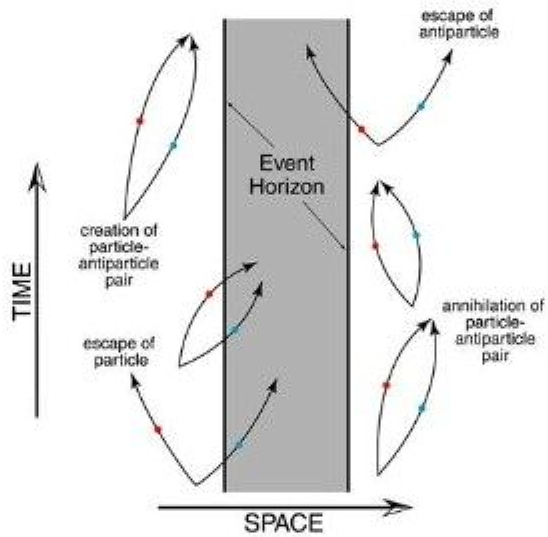


Accelerating Observer's Horizon

The observer's event horizon becomes a holographic screen when it encodes information along the lines of the holographic principle, with one bit of information encoded per pixel on the screen. The total number n of bits of information encoded is given in terms of the surface area A of the horizon and the Planck area, $\ell^2=\hbar G/c^3$, as $n=A/4\ell^2$.

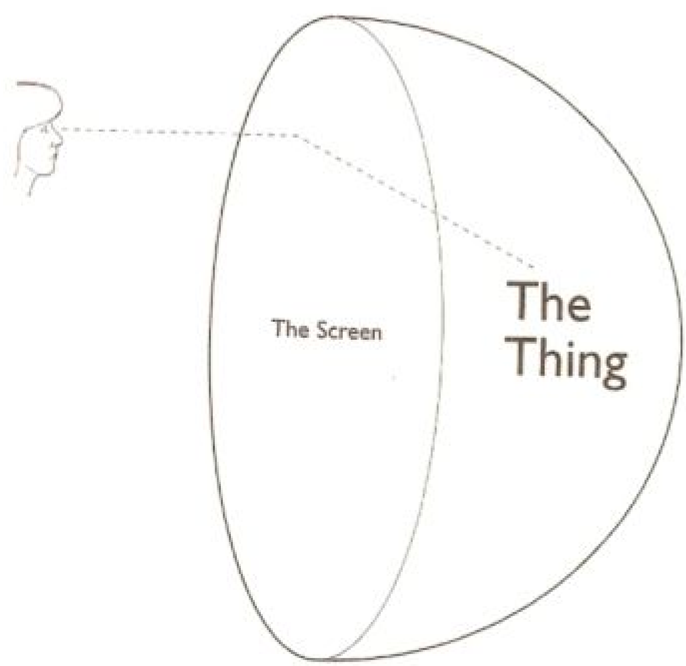


The observer's horizon not only encodes bits of information, but also has a temperature called an Unruh temperature, given in terms of the observer's acceleration a as $kT=\hbar a/2\pi c$. The Unruh temperature can be understood the same way Hawking radiation from the event horizon of a black hole is understood in terms of the separation of virtual particle-antiparticle pairs at the event horizon. As virtual particle-antiparticle pairs separate at the event horizon, an external observer will see the separated particles as a kind of thermal radiation, which corresponds to a temperature in the sense of blackbody radiation from a hot object.



Hawking Radiation

The second law of thermodynamics says that the energy E of a thermodynamic system is related to the entropy S and temperature T of that system as $\Delta E = T\Delta S$. The thermodynamic system of interest is whatever observable things the observer can observe in terms of the images of things projected from its own holographic screen to its own point of view.



In terms of the holographic principle, the entropy is defined in terms of the number of bits of information encoded on the observer's holographic screen as $S=kn=kA/4\ell^2$, and temperature is defined in terms of the observer's acceleration or Unruh temperature as $kT=\hbar a/2\pi c$. In the framework of the holographic principle, these quantities must satisfy the second law of thermodynamics $\Delta E=T\Delta S$. This is the fundamental equation that explains how mass energy arises. Recall that in the derivation of mass energy, we had to assume a force F acted on a mass, which resulted in the acceleration of the mass as $F=ma$, and also a change in the energy E of the mass due to the work done on the mass as that force acted over a distance Δx as $\Delta E=F\Delta x$.

The equation $\Delta E=F\Delta x$ is really the second law of thermodynamics $\Delta E=T\Delta S$ in disguise. The appearance of a mass accelerating through space over time is a holographic projection of the image of the mass from the observer's holographic screen to the observer's point of view. The image of the mass arises from bits of information encoded on the observer's holographic screen, which is the nature of entropy S, which through holographic projection gives the appearance of a mass at the position x in space. The temperature T of the holographic screen plays the role of the force F. Recall that the Unruh temperature is given in terms of the observer's acceleration as $kT=\hbar a/2\pi c$. This is where the acceleration comes from in the law of motion $F=ma$.

Where does $E=mc^2$ come from? The answer is a bit complicated, but has a holographic explanation. The equation $\Delta E=T\Delta S$ understood in terms of the holographic principle implies Einstein's field equations for the space-time metric, which is the law of gravity for whatever is observed in the space bounded by the observer's holographic screen. With the usual unification mechanisms of super-symmetry and the Kaluza-Klein mechanism, all the usual quantum fields of the standard model of particle physics arise from Einstein's field equations as extra components of the space-time metric. This includes the Higgs field, which is the mechanism that gives mass to all the elementary particles. In the Higgs mechanism, mass arises from the potential energy of the Higgs field through a process of spontaneous symmetry breaking, like the magnetization of a magnet when the temperature is low enough. In effect, all the bits of information are aligning together. The bits of information are like spin variables that can only point up or down, but act like little magnets that align when the temperature is low enough.

This alignment of the bits of information can be understood in terms of quantum entanglement, like entangled spin variables. In non-commutative geometry, the n bits of information encoded on a holographic screen are defined in terms of the n entangled eigenvalues of an SU(n) matrix, which behave like entangled spin variables that have a natural tendency to align together. This gives rise to the Higgs mechanism that gives all particles their mass. As previously mentioned, mass energy is a kind of potential energy that arises from the Higgs potential.

When we observe a particle with a mass m, we are really observing a holographic projection from a holographic screen. The image of the particle in space is projected from the observer's holographic screen to its point of view. That image is defined in terms of bits of information encoded on the screen. The screen arises because the observer is in an accelerated frame of reference. The observed particle has an energy because the observer's holographic screen has a temperature that arises from the observer's acceleration. Part of that energy is kinetic energy and part of that energy is potential energy. The mass energy is part of the potential energy.

There is a nice way to demonstrate $E=mc^2$ with the holographic principle for a black hole. The radius R of the event horizon of a black hole is defined in terms of its mass M as $R=2GM/c^2$. The acceleration due to gravity at the event horizon is $a=GM/R^2=c^2/2R$, which gives it an Unruh temperature of $kT=\hbar a/2\pi c=\hbar c/4\pi R$. The area of the event horizon is $A=4\pi R^2$, which gives an entropy $S=kA/4\ell^2$. If we write $\Delta E=T\Delta S$ in terms of $\Delta S=k\Delta A/4\ell^2$, $\Delta A=8\pi R\Delta R$, $\Delta R=2G\Delta M/c^2$, $\ell^2=\hbar G/c^3$, and $kT=\hbar c/4\pi R$, then $\Delta E=T\Delta S=(\hbar c/4\pi R)(8\pi R)(2G\Delta M/c^2)/4(\hbar G/c^3)$. All the factors cancel out except for $\Delta E=\Delta Mc^2$. By some kind of holographic miracle we have $E=Mc^2$.

In physics, a dynamical system is defined by specifying a set of degrees of freedom, and the laws of physics are set by specifying interactions among the degrees of freedom. The dynamical system always has a geometric representation, which is represented in terms of a space-time geometry. In particle physics, we assume a space-time geometry, take particle coordinates as the dynamical degrees of freedom, and assume local interactions among the particles in terms of a quantum field theory. The holographic principle tells us these assumptions are flat-out wrong.

With the holographic principle, the dynamical degrees of freedom are defined on a bounding surface of space. That is why the cosmological constant is a boundary condition. The observable physical universe as taken to be a dynamical system observed by the observer at the central point of view is defined on a de Sitter cosmic horizon. The observer's horizon is the boundary where all the degrees of freedom are defined. This is the boundary condition that defines the dynamical system. We also have to define the interactions. Non-commutative geometry defines the degrees of freedom as bits of information encoded on the horizon in terms of an SU(n) matrix and specifies the interactions in terms of the n entangled eigenvalues of that matrix. Due to the nature of entanglement, those boundary interactions are inherently non-local. Any local theory that emerges as a thermodynamic equation of state in a bounded region of space can at best be an approximate description of what appears to happen in that bounded space, but can never fully encompass the non-local way interactions occur on the bounding surface of that space.

The holographic principle is a radical departure from the concepts of both classical and quantum physics. In the classical concept of particle physics, the dynamical degrees of freedom of any bound or unbound state of particles observed in the world are described by particle coordinates, which define a phase space in terms of particle position and momentum variables. In quantum theory, particle position and momentum coordinates are represented by non-commuting variables that give rise to quantized values for particle position and momentum. These non-commuting variables give rise to the uncertainty principle, as the uncertainty in momentum, p, and the uncertainty in position, x, satisfy an uncertainty relation $\Delta p\Delta x \simeq \frac{1}{2}\hbar$. Forces between particles are represented by fields, like the gravitational and electromagnetic fields. With quantum field theory, even these force fields are understood to be composed of force particles like the photon or graviton that arise as localized wave-packets of field energy and momentum. The matter particles like the electron are also represented by quantum fields. With unification, all quantum fields are understood to arise as extra components of the space-time metric, which describes the dynamical nature of the space-time geometry of some bounded region of space. In this way, all degrees of freedom of any bounded region of space are represented by dynamical variables. Quantization of dynamical variables gives rise to the entropy of that bounded region of space.

The holographic principle is telling us that none of these classical or quantum concepts are really fundamental. Particle coordinates in any bounded region of space are not really fundamental dynamical variables. The way bits of information are encoded on the bounding surface of that space is the more fundamental description. Non-commutative geometry tells us the fundamental dynamical variables are non-commuting position coordinates on the bounding surface that are smeared out into area elements like pixels and encode bits of information in a binary code. The bits of information encoded on the bounding surface are the fundamental nature of entropy for whatever can be observed in that bounded region of space. Entropy is defined in terms of the number of all possible configuration states, Ω, as $S=k\log\Omega$. Since each bit of information on the bounding surface encodes information in a binary code of 1's and 0's, the number of all possible configuration states is $\Omega=2^n$, where the number of bits of information is given in terms of the surface area of the bounding surface as $n=A/4\ell^2$, which gives $S=kn$. This is a radical departure from the way entropy is described in either classical or quantum particle physics.

The amazing aspect of the holographic principle is it tells us this radical departure from the way entropy is described by particle physics in a bounded region of space is equivalent to the way entropy is more fundamentally defined in terms of bits of information encoded on the bounding surface of that space. This equivalence is due to holographic projection. The bounding surface arises as an observation-limiting event horizon in an observer's accelerated reference frame. The thermal energy of that bounded region of space arises from the observer's acceleration, which gives rise to the temperature of the bounding surface. Everything the observer can observe in the bounded region of space is like a holographic projection of images from the bounding surface, which acts as a holographic screen, to the observer's central point of view.

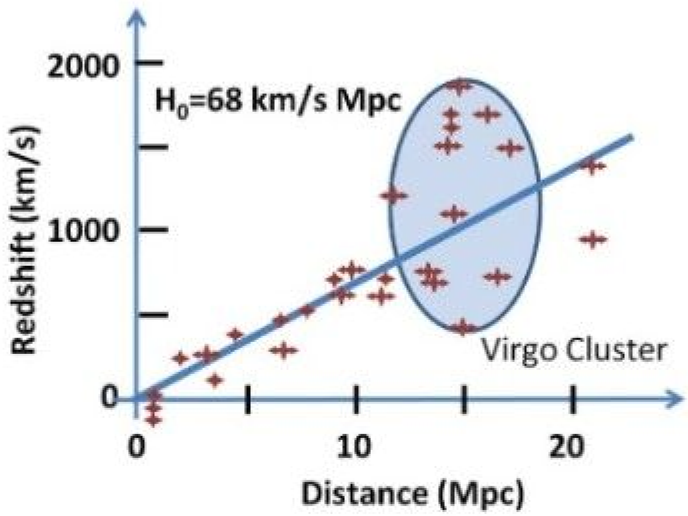
Remarkably, the holographic principle tells us that everything that can appear to happen from the point of view of an observer in any observable world, which is always a region of space that is bounded by a holographic screen that projects images of that world to the observer's central point of view, is as though nothing happens. It is as though nothing happens because all the energy for those happenings exactly adds up to zero. This is possible in relativity theory since the negative potential energy of gravitational attraction can exactly cancel out all forms of positive energy. A holographic world is fundamentally a world that is equivalent to nothing. A holographic world is also a conceptual world that consists of nothing more than forms of information projected like images from a screen to the point of view of an observer and animated in the flow of energy.

The remarkable recent discovery of modern cosmology is observations indicate the total energy of the observable universe is exactly zero. The fact the total energy of the observable universe exactly adds up to zero tells us something important. Since everything in the world is composed of energy and all energy ultimately adds up to zero, this tells us everything is ultimately nothing.

How do we understand that everything is ultimately nothing? The answer is found in all theories of the big bang event. All theories of the creation of the physical universe along the lines of the big bang assume the creative energy is dark energy, which is the accelerated expansion of space that always expands relative to the central point of view of an observer, which is the point of singularity of the big bang. In some mysterious way, the nothingness of the true vacuum state, which is like an empty space of potentiality, gives rise to the accelerated expansion of space. The positive energy of this accelerated expansion of space, which we call dark energy, is always exactly cancelled out by the negative potential energy of gravitational attraction, and so it is as though nothing ever happens. Everything ultimately adds up to zero.

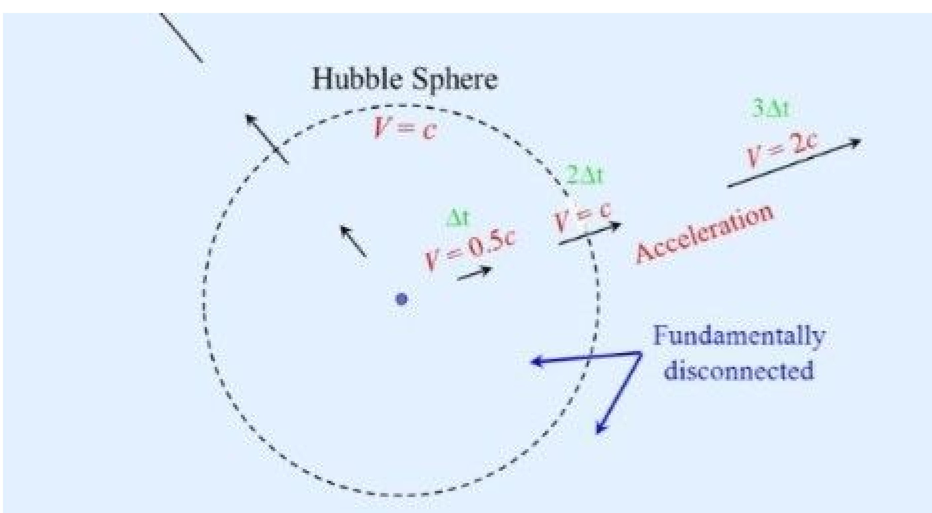
The accelerated expansion of space that arises from the expression of dark energy gives rise to a cosmic horizon that limits the observer's observation of things in space. Just as we understand the event horizon of a black hole in terms of thermodynamic equations, we can understand the thermodynamics of a cosmic horizon in a similar way. The first thing is to define the cosmic horizon. There are actually a number of more or less equivalent ways to define the cosmic horizon, such as with a cosmological constant or with Hubble's constant.

A world with dark energy is a world where space is expanding. This expansion of space is characterized by Hubble's law, which says things like galaxies move away from the central point of view of an observer with a velocity of movement that is proportion to the distance of separation. That's exactly what Hubble discovered when he measured the rate with which distant galaxies are moving away from the central point of the observer looking through the telescope. He had a way to measure the distance to galaxies in terms of the luminosities of certain stellar objects, and he had a way to measure their velocities in terms of the red shift of their spectra. He discovered Hubble's law, $v=H_0d$, where v is the velocity of a galaxy moving away from the observer, H_0 is Hubble's constant, and d is the distance of separation. Distant galaxies move away from the observer faster the farther out they appear in space, which is characteristic of an expanding universe. Hubble's constant is like an acceleration since it gives velocity in terms of distance, but unlike an acceleration in time, it's an acceleration in distance.



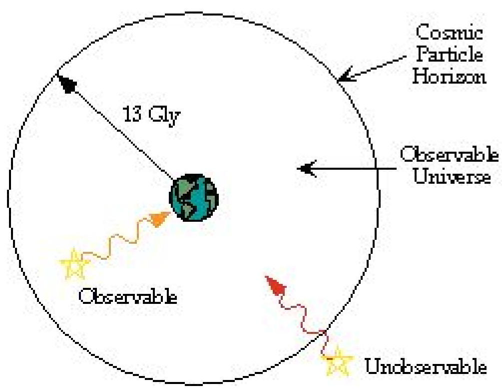
Hubble's Law

The cosmic horizon is defined by v=c. The cosmic horizon is a surface in space where objects in space appear to move away from the observer at the speed of light, and so nothing is observable beyond the cosmic horizon. The distance from the observer to the cosmic horizon is $d=c/H_0$. Since the observer is at the central point of view of its own cosmic horizon, this gives the radius of the cosmic horizon as $R=c/H_0$. This cosmic horizon defined in terms of Hubble's constant is also called the Hubble sphere. Just like the event horizon of a black hole, the cosmic horizon has a temperature given in terms of its radius as $kT=\hbar c/2\pi R$. If we interpret H_0 as an acceleration, this temperature takes the form of an Unruh temperature $kT=\hbar H_0/2\pi$. This thermodynamic description of the cosmic horizon is completed by an expression for its entropy $S=kn=kA/4\ell^2$, where A is the surface area $A=4\pi R^2$, and by the second law of thermodynamics $\Delta E=T\Delta S$.



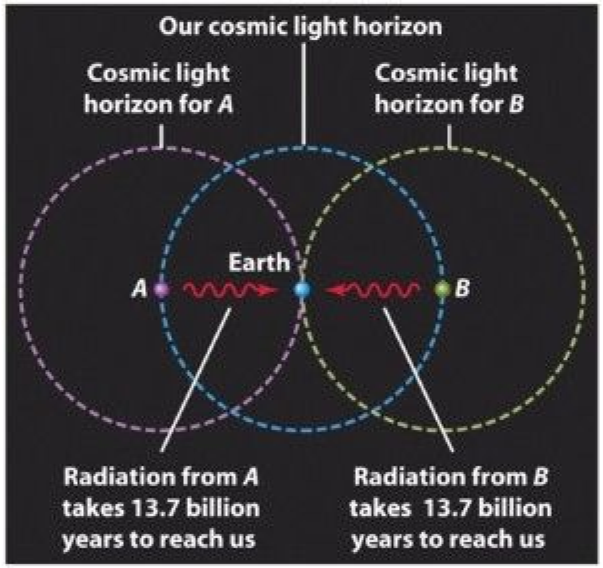
We can measure the rate with which distant galaxies appear to move away from us with a velocity $v=H_0d$ given in terms of the distance of separation according to Hubble's law, but we can also independently measure the rate with those galaxies are accelerating. The velocities could be slowing down or deaccelerating due to the attractive force of gravity, but it turns out the velocities are speeding up due to the repulsive force of dark energy. Dark energy is responsible for the acceleration, which we understand as a cosmological constant. The measurement of acceleration independent of velocity gives a value for the cosmological constant Λ. In Einstein's field equations for the space-time metric, the cosmological constant gives rise to the accelerated expansion of space. This results in a radius to the cosmic horizon expressed as $R^2=3\ell^2/\Lambda$. The measured value of Λ is about 10^{-123} based on the rate distant galaxies accelerate away from us.

We can also express the radius to the cosmic horizon in terms of Hubble's constant as $R=c/H_0$, where H_0 is measured to be about 20 kilometers per second per million light years. The speed of light c is about 3 x 10^5 km per second, and the Planck area ℓ^2 is about 2.6 x 10^{-66} cm^2. A light year is about 10^{18} centimeters, which is about 10^{51} Planck lengths, and so ℓ is about 10^{-51} light years and ℓ^2 is about 10^{-102} (light year)^2. The calculated value for the radius of the cosmic horizon from Hubble's constant, $R=c/H_0$, is about 15 billion light years. On the other hand, the calculated value for the radius of the cosmic horizon from the cosmological constant, $R^2=3\ell^2/\Lambda$, gives R as about 60 billion light years. These obviously don't agree. What is going on here?



Cosmic Particle Horizon

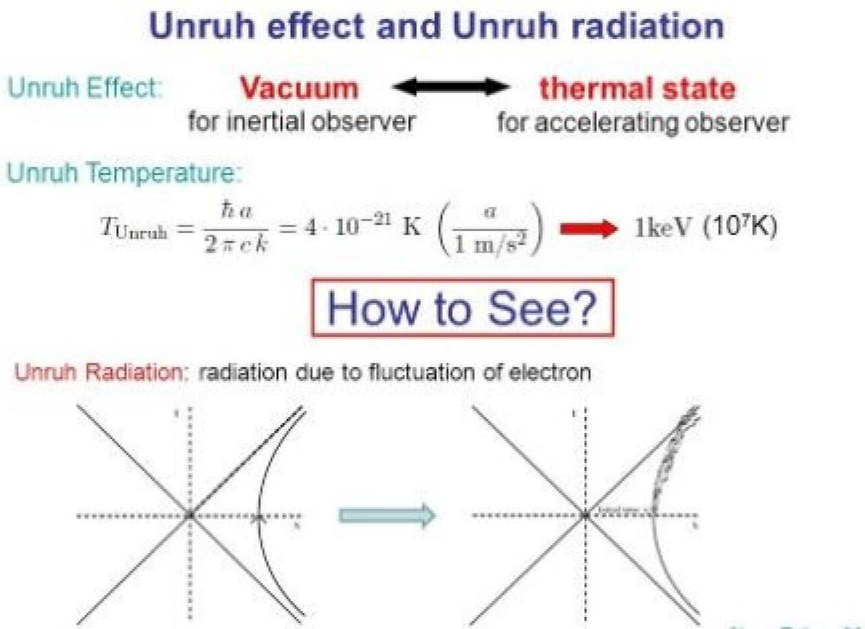
There is also another type of cosmic horizon called a particle horizon. The radius of the particle horizon is the maximal distance that light could have traveled in the past, or over the age of the universe, to reach the observer now. The radius of the particle horizon is about 40 billion light years since the expansion of space stretches out this distance. This value is more in line with the radius of a de Sitter cosmic horizon that corresponds to a cosmological constant of about 10^{-123}.



Overlapping Cosmic Light Horizons

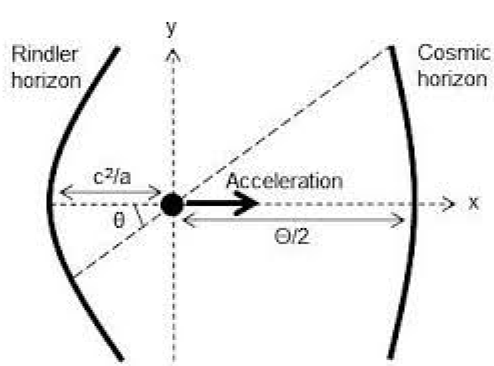
Why are these values different? The answer is a cosmic horizon is an event horizon that arises in an observer's accelerated reference frame. The distance to the event horizon depends on how the observer accelerates. The horizon is observer-dependent. Just as the distance to the event horizon depends on how the observer accelerates, so too is everything the observer can possibly observe in its world, since the observer's horizon limits the observer's observations of things in space.

The reason these values can be different is because the distance from the observer to its event horizon can be any distance. The classic example of this effect is a Rindler horizon, which is an event horizon that arises for an observer that undergoes constant acceleration. The observer's horizon is entirely due to the observer's acceleration. If the observer accelerates in a different way, the observer has a different horizon. The Unruh temperature is derived as the temperature of a Rindler horizon, which is $kT=\hbar a/2\pi c$ when the observer accelerates with an acceleration a.



Unruh Radiation from a Rindler Horizon

When we discuss the radius of a cosmic horizon, we have to specify the observer's acceleration, since the distance to the observer's horizon depends on that acceleration. Every possible horizon is observer-dependent, and can only arise in the observer's accelerated frame of reference. Just as the observer's horizon only arises if the observer accelerates, everything the observer can observe in space also depends on the observer's acceleration. The holographic principle tells us all the fundamental bits of information that define all the observable things in space are encoded on the observer's horizon, which acts as a holographic screen. Things are only observed in space when forms of information are projected like images from the observer's holographic screen to its central point of view. The nature of observation is holographic projection, and the observation of things can only occur when the observer is in an accelerated frame of reference.

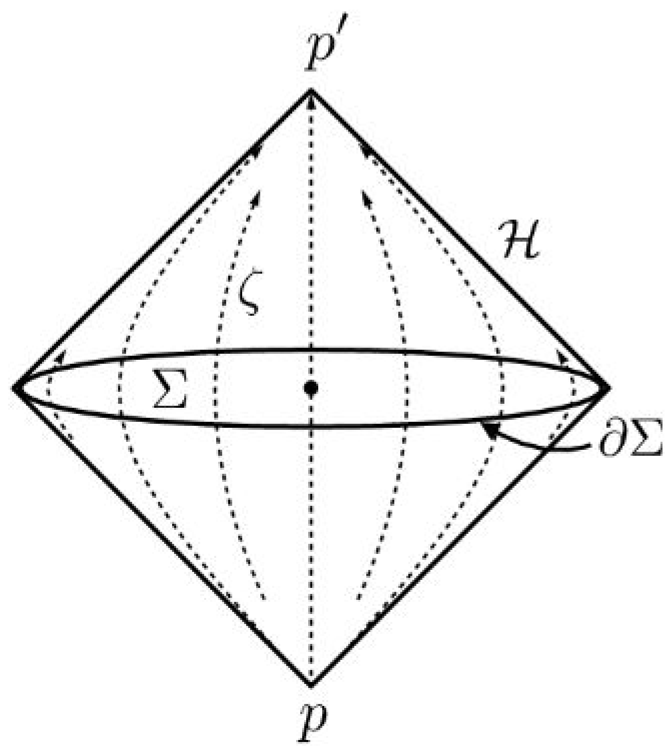


Rindler versus Cosmic Horizon

When we observe something, like a point particle that follows a trajectory through space, we are actually observing a sequence of holographic screens, each of which encodes information for the object at some moment of time. Those holographic screens can be layered together, so that each observation of a point on the trajectory corresponds to observing how that point pierces another layer of the screens. Holographic screens are inherently two dimensional, but the layering of screens gives the illusion of three dimensional space. Like all event horizons that arise in an accelerated reference frame, the distance to the screen is set by the observer's acceleration.

As already pointed out, the cosmological constant is a boundary condition that sets the boundary for the observable physical universe, which is always observed by the observer at the central point of view. All the dynamical degrees of freedom for whatever the observer observes in that world are defined on the boundary. The way bits of information are encoded and interact on the boundary is inherently non-local since interactions arise though quantum entanglement, typically as entangled eigenvalues of an SU(n) matrix. Although things appear to be local and appear to locally interact within the bounded region of space, this is an illusion of holographic projection, which is inherently non-local. When we trace the trajectory of a point particle through space over time as that point pierces through the layering of many holographic screens, the way information is encoded on those screens for the particle and its interactions is inherently non-local.

This way of formulating the holographic principle in terms of the layering of holographic screens has a natural representation in relativity theory in terms of the concept of a causal diamond. The idea of a causal diamond is related to the idea of an observer's light cone. From the perspective of any observer's point of view, the observer's past and future light cones represent causal events that can be observed in the past and in the future, which are connected by light rays. The causal diamond is constructed by connecting events the observer can observe in the past to events the observer can observe in the future. At the center of the causal diamond is a spherical surface where past events are connected to future events through the connection of light rays.



Observer's Causal Diamond Representation of Past and Future Events

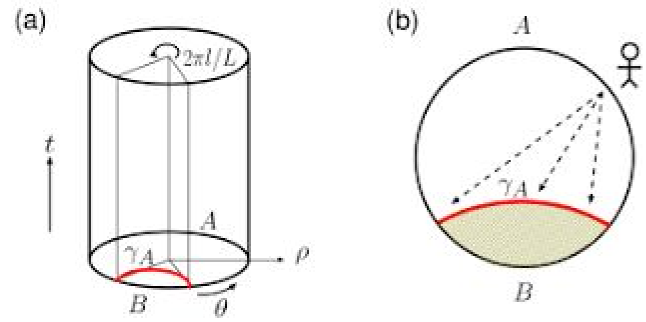
The observer's causal diamond connects past events to future events. At the center of the causal diamond is a spherical surface with the properties of an event horizon. This spherical surface connects events that can be observed in the past to events that can be observed in the future. This bounding surface becomes a holographic screen when it encodes information along the lines of the holographic principle, which it naturally does when a finite number of position coordinates are defined on the surface in terms of non-commuting variables and non-commutative geometry. Whatever forms of information the observer observes in its causal diamond are then the result of holographic projection from this holographic screen to the observer's central point of view.

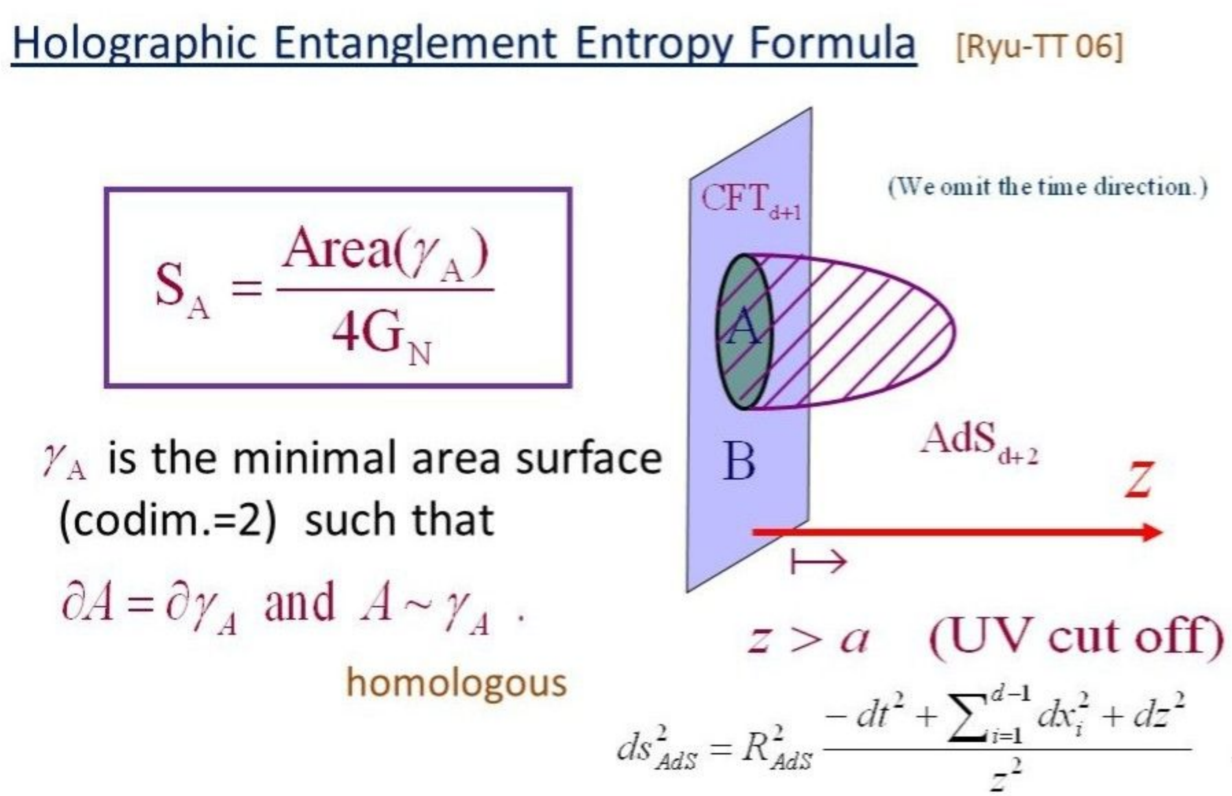
A great deal of confusion arises when the holographic principle is analyzed in the framework of quantum field theory. This confusion results from assuming the holographic principle can be understood in terms of quantum field theory, but it cannot. The first problem is the assumption of locality. Quantum field theory is a local theory that localizes particles in space-time geometry. This is reflected by the local symmetries of quantum field theory, like time translation invariance and spatial translation invariance. These are symmetries of flat Minkowski space. The problem is the observable physical universe is not characterized by flat Minkowski space, but by de Sitter space. In an exponentially expanding universe that begins in a singular big bang event, there is no such thing as time translation invariance. With the accelerated expansion of space, every observer is surrounded by an observation-limiting cosmic horizon and there is no such thing as spatial translation invariance. These local symmetries do not apply to the observable universe.

The second problem is quantum field theory is at best an effective field theory only valid near thermal equilibrium, like a thermodynamic equation of state. Even Einstein's field equations for the space-time metric only have the validity of thermodynamic equations of state. It only makes sense to quantize an effective field theory for small fluctuations around the vacuum state. For larger fluctuations, the effective field theory only has the validity of a thermodynamic equation of state, like the wave equation for sound waves. The wave equation for sound waves can be quantized in terms of particle excitations called phonons, but this only has validity near the vacuum state. The particle excitations of the electromagnetic field, which are called photons, or of the gravitational field, which are called gravitons, are no more fundamental than phonons. Underlying these effective field theories is a more fundamental microscopic theory, which for sound waves is atomic theory and for quantum field theory is the holographic principle.

Confusion about the holographic principle arises when quantum field theory is used to analyze it, but this assumption is not valid. An example of this confusion is the firewall paradox at the event horizon of a black hole, which is a variation of the information loss paradox. Hawking radiation is understood to arise from separation of virtual particle-antiparticle pairs at the event horizon as observed by an external observer. The virtual pairs are entangled and their separation gives rise to the temperature and entropy of the event horizon, which is understood as an entanglement entropy. The firewall paradox arises when Hawking radiation that occurs later in time is required to be entangled with earlier Hawking radiation to insure there is no loss of information. In quantum field theory, entanglement entropy is additive, and so virtual particle-antiparticle pairs cannot be entangled if the late and early Hawking radiation are also entangled. If the virtual pairs are not entangled, an observer that falls across the event horizon will see a hot firewall, but that violates the principle of equivalence, which says the event horizon is only an imaginary boundary in space for a freely falling observer. The firewall paradox says that either local quantum field theory must be violated or the equivalence principle must be violated. The solution to this paradox is to abandon the idea of local quantum field theory and what it tells us about entanglement entropy. Non-commutative geometry solves this paradox since it tells us the way entropy is encoded on any bounding surface of space is inherently non-local.

The non-local way entropy is encoded on a bounding surface of space that acts as a holographic screen is demonstrated in the AdS/CFT correspondence. The conformal field theory defined on the boundary of anti-de Sitter space has degrees of freedom that act like spin variables, and so the entropy of any state defined on the boundary can be defined. All the degrees of freedom of the entire boundary are entangled, and so information is encoded on the boundary in a non-local way. When the boundary is divided into two separate regions A and B, the dynamical degrees of freedom of the two regions remain entangled like entangled spin variables that are separated. An entanglement entropy characterizing each region of the boundary can be defined that describes the missing information of the other region, which is needed to describe each region since all the information is entangled. This entanglement entropy corresponds to a surface of minimal area that is like a geodesic path through anti-de Sitter space that connects the endpoints of each region of the boundary. This minimal area surface through anti-de Sitter space gives an exact measure of the entropy of the corresponding region of the boundary in terms of the holographic principle. The entanglement entropy of the boundary region is given in terms of the surface area A of the minimal area surface through anti-de Sitter space as $S=kA/4\ell^2$, where the Planck area is defined in terms of the gravitational constant $\ell^2=\hbar G/c^3$ that corresponds to gravity in anti-de Sitter space.





Entanglement Entropy of the Minimal Area Surface

The conformal field theory defined on the boundary has no description of gravity, and yet gravity somehow emerges in anti-de Sitter space from the way entangled bits of information are encoded on the boundary. How does this happen? The answer of course is holography. The entangled bits of information of the conformal field theory defined on the boundary give rise to the emergence of gravity in anti-de Sitter space through holographic projection. The minimal area surface through anti-de Sitter space is acting as a holographic screen.

The minimal area surface through anti-de Sitter space connects endpoints of the boundary region and acts like a holographic screen for whatever is observed in a region of anti-de Sitter space. That region of anti-de Sitter space is bounded by both the minimal area surface and the boundary region. This is just like a holographic screen that arises in an observer's causal diamond. All the information for whatever is observed in that region of anti-de Sitter space is entangled on the entire boundary of anti-de Sitter space in a non-local way, which gives rise to the entanglement entropy of the boundary region. The conformal field theory defined on the boundary plays the role of non-commutative geometry in terms of defining how all the bits of information defined on the boundary are entangled like the entangled eigenvalues of an SU(n) matrix. The entropy of the minimal area surface through anti-de Sitter space that acts as a holographic screen is equal to the entanglement entropy of the boundary region, which gives an explicit demonstration of the holographic principle in the AdS/CFT correspondence.

Entanglement entropy can be seen in as simple a system as two entangled spin variables. A spin variable encodes two bits of information in terms of spin up or spin down, or spin right or spin left circular polarization. These two spin states can be written in terms of a bit of information as [1> or [0>. A qubit is defined as a linear superposition [ψ>=a[1>+b[0>. Since the spin variable must be either up or down, the probability amplitudes satisfy $a^2+b^2=1$. The total probability must be unity. When two spins become entangled, an entangled state of oppositely directed spins can be written as [ψ>=a[10>+b[01>. Even when the spin variables are separated, measurement of the spin state of either spin determines the spin state of the other spin, but when unmeasured, we can only speak about the probability of how those unmeasured spin states can be measured.

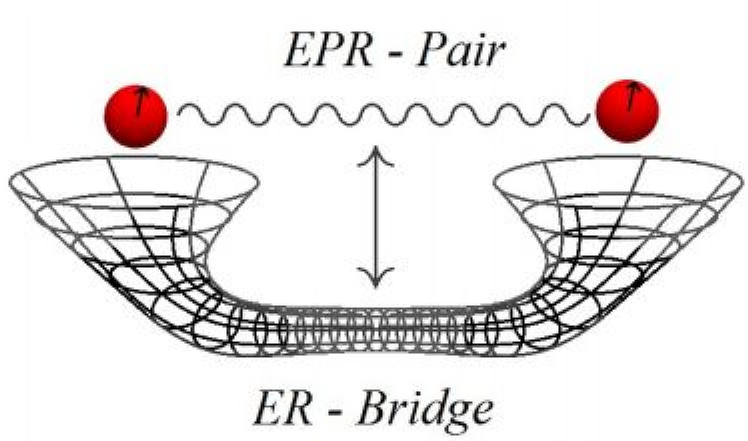
For example, if $a^2=99/100$ and $b^2=1/100$, we can say with 99% certainty that the first spin will be measured up and the second spin will be measured down, but there is still 1% uncertainty that things will be measured the other way around. Entanglement entropy is telling us something about these probabilities of measurement even when the spin variables become separated. We can never speak about an independent probability of measurement of each spin variable alone when the spin states are entangled. Entanglement entropy is a way of describing the information state of each spin variable in terms of the missing information of the entangled partner even when the spin variables become separated.

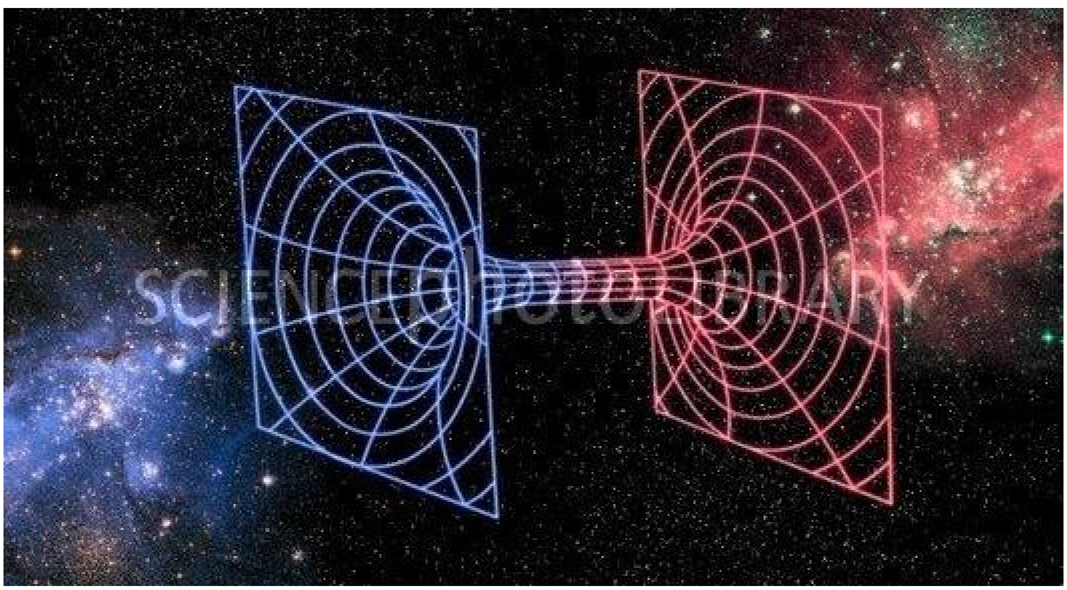
The holographic principle is understood in the AdS/CFT correspondence in terms of entangled bits of information encoded by a conformal field theory on the boundary of anti-de Sitter space. The entanglement entropy of separate regions of the anti-de Sitter space boundary give rise to holographic screens that arise in anti-de Sitter space as minimal area surfaces. Much like the formulation of the holographic principle with non-commuting variables defined on a bounding surface of space, the conformal field theory plays the role of separated entangled spin variables that encode entangled bits of information on a bounding surface of space.

This way of understanding the holographic principle in terms of entangled bits of information on a bounding surface has implications for the nature of space-time geometry. If we separate a large number of entangled spin variables and then collapse those separated spin variables into two separate black holes, those two black holes remain connected through their entanglement. The nature of this connection is purely geometrical. The two connected black holes are connected by a wormhole. The geometric structure of the wormhole is the nature of entanglement.

In the AdS/CFT correspondence, two black holes connected by a wormhole in anti-de Sitter space arise from entanglement of bits of information encoded by the conformal field theory on the boundary of anti-de Sitter space. This is just like the collapse of separated entangled spin variables into two separate black holes. The two black holes are geometrically connected by a wormhole, but the geometric structure of the wormhole arises from the entanglement of the separated bits of information. This is an explicit example of how entanglement of information on a bounding surface of space gives rise to the geometric structure of space-time geometry.

This example of space-time geometry emerging from the entanglement of pure information encoded on a bounding surface of space is called ER=EPR. ER stands for Einstein Rosen, which is the classic paper that describes a wormhole between two black holes. The wormhole is called an Einstein-Rosen bridge. EPR stands for Einstein, Podolsky and Rosen, which is the classic paper that describes quantum entanglement. Entanglement of two spin variables is called an EPR pair. The AdS/CFT correspondence demonstrates that the entanglement of bits of information encoded on a bounding surface of space, like an EPR pair, gives rise to geometric structures within that bounded space-time geometry, like a wormhole between two black holes.

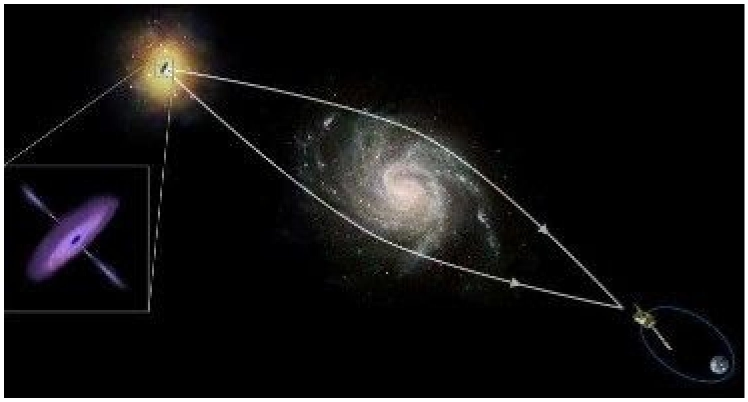




Black Holes Connected by a Wormhole due to Entanglement of EPR Pairs

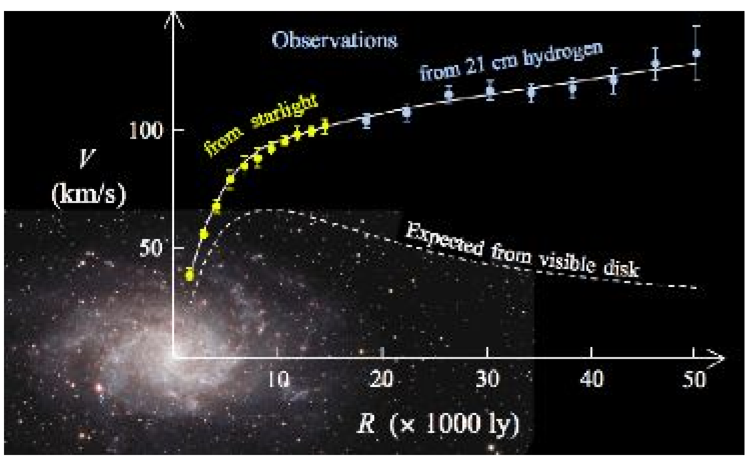
This discovery that the entanglement of information on a bounding surface of space gives rise to geometric structures within a bounded space-time geometry is called holographic emergence. The basic idea is that space-time geometry emerges from an entangled state of pure information. Since gravity is nothing more than the curvature of space-time geometry, it is then possible to say that gravity emerges from an entangled state of pure information. The holographic principle is a way of mathematically formulating this kind of holographic emergence.

In recent years, a number of theoretical physicists, most prominently Erik Verlinde, have tried to extend the AdS/CFT correspondence into de Sitter space in an attempt to develop a theory of emergent gravity in de Sitter space. They hypothesize that anti-de Sitter space is the ground state of de Sitter space, and that dark energy is the energy of excitation that excites de Sitter space from anti-de Sitter space. The theory of gravity that emerges is interesting in two ways, as it not only deviates from Einstein's theory of gravity at the short distance scale of the Planck length but also deviates from Einstein's theory at the long distance scale of the cosmological scale.



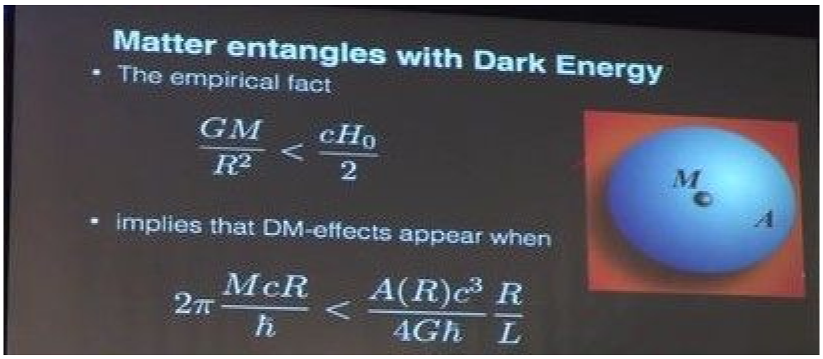
Gravitational Lensing

The motivation for a theory of emergent gravity has to do with the anomalous behavior of dark matter. Einstein's law of gravity seems to break down for certain galactic phenomena, like the gravitational lensing of light around galaxies and the rotation of galaxies. Einstein's theory predicts a certain velocity of rotation of stars around the edges of galaxies based on the total amount of visible mass seen in the galactic centers. The problem is the rotational velocities are too large given the amount of visible mass. To solve this problem, astronomers have proposed there is an invisible halo of dark matter that surrounds each galaxy, which allows the rotational velocities to be larger than expected. The distribution of dark matter has to be put in by hand and there is no fundamental way to explain it or even to directly measure it.

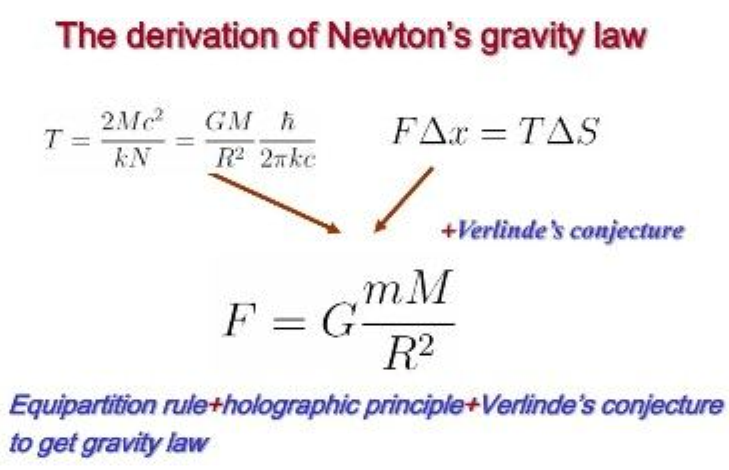


Deviation of Galactic Rotational Velocities from Expected Result

There is an anomalous aspect to the dark matter effect than Einstein's theory cannot explain. The deviation of the rotational velocities from the expected result always occurs at a certain point when the acceleration of gravity is related to Hubble's constant in a very specific way. This is weird since there is no reason why the gravity generated by the mass of a galaxy should have anything to do with Hubble's law, but the evidence is that it does. The deviation in rotational velocities around the edge of the galaxy always occurs when the acceleration of gravity due to the visible mass of the galaxy is approximately equal to an acceleration due to Hubble's constant as $GM/R^2 \approxeq cH_0/2$. Why should dark matter have anything to do with Hubble's law?



The answer Verlinde gives with a theory of emergent gravity is dark matter does not really exist. The apparent effect of dark matter is due to the dynamics of dark energy. This is understood in the sense of the holographic principle and the encoding of bits of information on a bounding surface of space, which is taken to be a de Sitter cosmic horizon. Using the area law for entropy $S=kn=kA/4\ell^2$, given in terms of the surface area of the horizon, along with the Unruh formula for the temperature of the horizon $kT=\hbar g/2\pi c$, where the acceleration at the horizon can be written in terms of Hubble's constant as $g=cH_0$ and $H_0=c/R$ is given in terms of the horizon's radius, Einstein's field equations for gravity can be shown to emerge from this holographic formulation as thermodynamic equations of state, which Verlinde calls an entropic force.



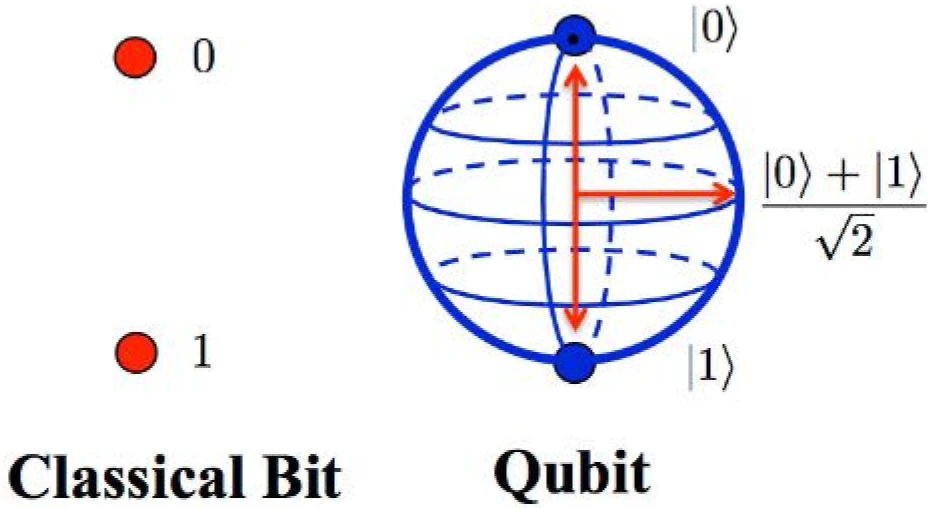
Verlinde extends this idea to include volume effects of dark energy in addition to area effects. The basic idea is to think of dark energy in de Sitter space as an excitation of anti-de Sitter space.

The AdS/CFT correspondence tells us there is only an area law for entropy that arises from entangled bits of information encoded on the anti-de Sitter horizon. Verlinde hypothesizes that in de Sitter space there is a volume component of entropy. This volume component is expressed in terms of qubits or entangled quantum bits of information that occupy the volume of de Sitter space. These qubits of de Sitter space represent the entropy of dark energy as an excitation of anti-de Sitter space. The entangled qubits have long range effects that arise from large distances of entanglement, and also have a kind of dynamics similar to the dynamics of long polymers that are composed of a large number of atoms bound together into long chain-like linear structures. The basic idea is the qubits are the atoms and entanglement is the nature of the atomic bonds.

Qubits Entangled with the de Sitter Horizon and Entangled Together in de Sitter Space

Einstein's theory of gravity arises from the entropy encoded on the de Sitter horizon according to the area law, but the volume component of entropy also gives rise to emergent effects of gravity that deviate from Einstein's theory at large distances where the cosmological effect of Hubble's law cannot be ignored. When entangled qubits connect to the de Sitter cosmic horizon, entropy follows an area law, but when entangled qubits connect together within de Sitter space, there is also a volume contribution to the entropy. These volume effects of dark energy entropy have a dynamics that alters the usual law of gravity, just as observed for the rotational velocities of galaxies or with gravitational lensing. In this picture, the apparent effects of dark matter are entirely due to the long range dynamics of dark energy. The equations that result from these ideas completely explain all the apparent dark matter effects with no free parameters.

What are the qubits Verlinde uses in his calculation? The best way to think of the qubits are as Planck size black holes that encode two bits of information. The event horizon of a Planck size black hole can only encode two bits of information, which are represented by an SU(2) matrix. The SU(2) matrix is describing rotational symmetry on the surface of the event horizon, but is also the fundamental way two bits of information are defined in quantum theory as a qubit. For a Planck size black hole, n=2 in the area law for entropy $n=A/4\ell^2$, which is represented by an SU(2) matrix that encodes two bits of information as a [1> or as a [0>, but due to rotational symmetry on the surface of the event horizon, these bits of information can be represented as a quantum superposition over these two states [ψ>=a[1>+b[0>. This equation defines a qubit.



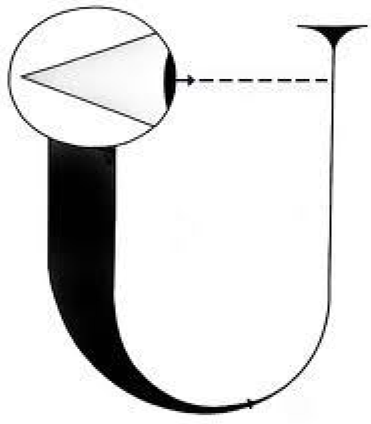
Qubit as the Information Encoded on the Event Horizon of a Planck Size Black Hole

If we think of the qubits as Planck size black holes, then what is quantum entanglement? The answer has recently been discovered in the AdS/CFT correspondence, and is referred to by the slogan ER=EPR. In 1935 Einstein wrote two famous papers with collaborators that at first glance seem totally unrelated, but now are the foundation for understanding the nature of dark energy, which ironically he called his biggest blunder when he proposed adding a cosmological constant term to his field equations. How weird is that? EPR stands for Einstein, Podolsky and Rosen, which is the classic paper that describes quantum entanglement. ER stands for Einstein Rosen, which is the classic paper that describes a wormhole or Einstein-Rosen bridge between two black holes. With our modern understanding of dark energy, a qubit in de Sitter space is a Planck size black hole that is connected by a wormhole. The wormhole is the nature of entanglement. The qubit can connect to other qubits in de Sitter space or to the de Sitter event horizon.

What's wrong with this picture? The answer is nothing is wrong if you're looking for an effective theory like a thermodynamic equation of state that explains the data. On the other hand, if you're looking for a fundamental explanation along the lines of the holographic principle, these ideas can at best be some kind of hybridization of fundamental ideas with ideas of emergent gravity.

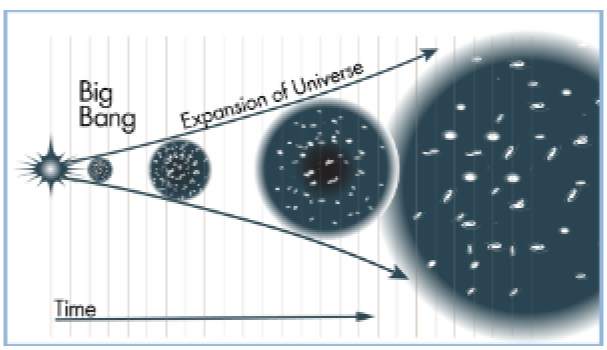
What are the fundamental ideas? There are only two. The first is an observer-centric description of the observable world. The second is holographic projection as the nature of observation.

The holographic principle is a confirmation of what John Wheeler called *It from bit*, which more recently has been called *It from Qubit*, referring to the entangled nature of quantum information. At a fundamental level, the observable universe is encoded in terms of entangled bits of information on a holographic screen, and is observed through holographic projection of forms of information projected like images from the screen to the central point of view of an observer.



Universal Observer

There are a number of puzzles about cosmology that even the holographic principle cannot explain. If we accept the world is created in a big bang event, we have to address the issue of how the energy for that world is created and also address the issue of how the information for that world is created. The issue of energy is the easier issue to address. All theories of the big bang assume some sort of dark energy, which is the energy that drives the accelerated expansion of space, is the driving force for the big bang event. Dark energy is the explosive force that puts the *bang* in the big bang event. Dark energy is the repulsive energy of the accelerated expansion of space, which like a kind of anti-gravity, throws everything out in a big explosion.



The Big Bang Event as the Accelerated Expansion of Space

Theories of the big bang event assume dark energy had a maximal value early during the history of the universe, but this value then transitions to a smaller value as the universe expands in size. It is this transition to a smaller value that is difficult to explain in terms of a natural mechanism. The best idea is that the value of dark energy is set by some kind of potential that has within it a number of metastable states. A metastable state is like a false vacuum, where the vacuum is understood as the lowest possible energy state. The natural vacuum energy is zero since that is the lowest possible value, while a metastable state has a non-zero vacuum energy, which makes it a false vacuum. This metatable state is like a ledge on the side of a mountain that has a higher gravitational potential energy than the ground level below it. When things are stuck on the ledge, they don't fall to the ground unless something pushes them off the ledge.

The basic idea is that a high value for dark energy early in the history of the universe is a metastable state that eventually must transition to a lower energy state. This transition is often thought of as a phase transition in which the universe transitions from a higher level metastable state of dark energy to lower energy state of dark energy. Since dark energy gives rise to a cosmic horizon with a radius that is inversely related to the value of dark energy, as dark energy transitions to a lower level, the radius of the cosmic horizon increases in size and the observable universe expands in size. Since the temperature of the cosmic horizon is inversely related to its radius, this temperature decreases as this radius increases and the observable universe expands in size. This decrease in temperature of the cosmic horizon is what gives rise to the thermal gradient within which heat flows as the observable universe expands in size. This flow of heat in a thermal gradient is what in turn gives rise to *time's arrow* and the natural flow of time.

As dark energy transitions to a lower value and the observable universe expands in size, the cosmic horizon increases in surface area and encodes more information. As the cosmic horizon cools in temperature and heat flow in this thermal gradient, entropy increases since information is created as the cosmic horizon increases in surface area and encodes more information. The expansion of the universe is therefore correlated with this increase in entropy and the normal flow of heat that occurs as the observable universe expands in size and cools in temperature.

The problem with this scenario is we don't have a good idea for a mechanism that gives rise to the metastable states that allow dark energy to transition to a lower value. Everything hinges on the value of dark energy. Just as a high value for dark energy is the explosive force that puts the *bang* in the big bang event, a decrease in the value of dark energy is what drives the expansion of the observable universe as the cosmic horizon increases in size. As the cosmic horizon increases in surface area, information for the observable universe is created as the horizon encodes more information. As the cosmic horizon increases in size, its temperature cools, which creates the thermal gradient within which heat flows. The normal course of time arises with the flow of heat. Everything about the natural evolution of the observable universe hinges on dark energy transitioning from an initial high value to lower values over the course of time, but we really don't have a natural mechanism to explain these transitions. The problem with the potential energy metastable state scenario is that we have to put these metastable states in by hand.

If we take the big bang event seriously, we understand that at the moment of creation of the observable universe a great deal of dark energy is expended. That world is initially only about a Planck length in size, but then expands in size due to an instability in the amount of dark energy.

This instability in dark energy is like a process that burns away dark energy. The big bang hypothesizes that at the moment of creation dark energy takes on a maximal value, but due to an instability in the amount of dark energy, the value transitions to a lower value. This transition is like a phase transition from a metastable false vacuum state to a more stable vacuum state of lower energy. The most stable state, the true vacuum state, is a state with zero dark energy.

The instability in dark energy is like a process that burns away dark energy. The expenditure of dark energy breaks the symmetry of empty space by constructing an observation limiting cosmic horizon that surrounds the observer at the central point of view. As dark energy burns away to zero, the cosmic horizon inflates in size to infinity and the symmetry is restored. We understand this undoing of symmetry breaking is like a phase transition from a false vacuum state to a true vacuum state. As the phase transition occurs, dark energy burns away and heat is radiated away. This idea is also consistent with the current measured value of dark energy, based on the rate with which distant galaxies are observed to accelerate away from us, which gives the size of the observable universe in light years as the same order of magnitude as the age of the universe.

This burning away of dark energy also explains the normal flow of energy in the observer's world in terms of the second law of thermodynamics. Relativity theory tells us the radius, R, of the observer's cosmic horizon is inversely related to the density of dark energy, while the holographic principle tells us the absolute temperature of the observer's horizon is inversely related to its radius. At the moment of creation, R is about a Planck length and the absolute temperature is maximal. As dark energy burns away to zero, R inflates in size to infinity, and the temperature cools to absolute zero, which is called the heat death of the universe. The second law simply says that heat tends to flow from hotter to colder objects because hotter objects radiate away more heat as thermal radiation. The instability in dark energy explains the second law as dark energy burns away, the observer's world inflates in size and cools in temperature, and as heat tends to flow from hotter states to colder states of the observer's world. This normal flow of energy through the observer's world naturally arises in the thermal gradient created by an expanding universe. This also explains the mystery of *time's arrow*, as the normal course of time is related to the normal flow of energy through the observer's world. As far as the holographic principle goes, a thermal gradient is also a temporal gradient. Time flows because heat flows.

How do other forms of energy, like mass energy, arise from dark energy? The answer is symmetry breaking. All forms of positive energy arise from dark energy through symmetry breaking. As dark energy burns away, high energy photons are created, and these photons can create particle-antiparticle pairs, like proton-antiproton pairs. One of the mysteries of cosmology is why there are so many protons in the universe and so few antiprotons. Symmetry breaking gives the answer. At high energies, protons can decay into positrons and anti-protons into electrons, but there is a difference in the decay rates due to a broken symmetry called parity violation, and so more antiprotons decay than protons. As the universe cools, protons become relatively stable, and so that's what's left over. Even the mass of the proton arises through a process of symmetry breaking called the Higgs mechanism. The expenditure of energy that characterizes all the gauge forces, like electromagnetic energy in a living organism or nuclear energy in a star, all arise from dark energy through a process of symmetry breaking, but all of this positive energy is exactly cancelled out by the negative potential energy of gravitational attraction, and so in the end, it is as though nothing ever happens.

The normal flow of energy through the observer's world is a consequence of the second law of thermodynamics, which describes the random flow of thermal energy. Heat tends to flow from hotter to colder objects, and also from hotter states to colder states of the observer's world. The observer's world is not at thermal equilibrium, which is why heat flows. This is purely a statistical consequence of hotter objects tending to radiate away more heat. As heat flows in a thermal gradient, entropy, which is the disordering of information inherent in objects as a consequence of the randomization of thermal energy, tends to increase, which tends to disorder objects, like a piece of ice that becomes more disordered when it melts into water as heat flows into it and chemical bonds are broken, or the flow of heat from the sun to the earth which arises through the dispersion of photons into more randomized states. This normal flow of heat in a thermal gradient and the corresponding increase in entropy that accompanies the flow of heat is what gives rise to the normal flow of energy through the observer's world.

Normal Flow of Energy

There is a competing process that tends to balance out the normal increase in entropy or disorder that occurs as heat flows in a thermal gradient. This balancing process is the tendency for coherent organization of information to develop, which allows for the organization of objects into distinct forms that coherently self-replicate their forms and for distinct forms to become inter-related. This tendency for coherent organization to develop is a natural aspect of a holographic world, since all the bits of information encoded on a holographic screen are entangled, typically as the n entangled eigenvalues of an SU(n) matrix that arise when a finite number of position coordinates are specified on the screen by non-commutative geometry. Entanglement of information implies that every distinct form of information that appears in the observer's world through the projection of images from the observer's holographic screen to its central point of view is inherently related to every other distinct form of information. Entangled bits of information naturally tend to align over an animated sequence of holographic projections, and that alignment of information gives rise to the coherent organization of information. Coherence can even be seen on a piece of holographic film as an interference pattern.

This tendency for entangled bits of information to align or bind together is typical of quantum entanglement, like entangled spin variables that tend to align. This tendency for entangled spin variables to align over a sequence of quantum state reductions is demonstrated in a spin network. A holographic screen has that kind of underlying structure. This natural tendency for entangled bits of information to self-organize and form self-replicating distinct forms of information and for the development of inter-relationships between distinct forms is balanced out by the natural tendency for information to become disordered and entropy to increase as heat flows in a thermal gradient. The temporary and local organization of information into forms is possible in spite of the relentless tendency for entropy to globally increase and eventually disorganize all forms due to the possibility of entropy locally and temporarily decreasing while global entropy increases.

This local and temporary decrease in entropy is possible due to the addition of organizing potential energy to a form while disorganizing random kinetic energy or heat is radiated away from the form into the global environment. We call this addition of organizing potential energy to a form the process of forms eating other forms. The necessity for a life-form to add potential energy to itself by eating other life-forms is a necessary condition for the temporary organization of all life-forms. Life-forms can only survive as self-replicating forms if they eat other life-forms and avoid being eaten by other life-forms. This kind of energetic expression by a life-form is what we call an emotional expression, as in the expression of fear and desire, which is necessary for survival of life-forms. Even plants have to eat photons through the process of photosynthesis. The organization and disorganization of the forms of all objects and their inter-relationships are always in a balanced state of interplay in a holographic world.

The ability of a coherently organized life-form to self-replicate its form is inherently dependent on emotional expressions. The only way life-forms can self-replicate their forms or survive in a recognizable form over a sequence of perceivable events is if they express emotions of fear and desire. Life-forms are only able to maintain their coherent organization if they add organizing potential energy to their forms through a process called eating. A life-form must have a source of organizing potential energy from which it feeds or adds energy to its form. For many life-forms, this process of adding potential energy to its form means the life-form must eat other life-forms.

The need to add organizing potential energy to a form through a process of eating in order to maintain the state of organization of that form in a recognizable form is a direct consequence of the disorganization of forms that occurs as heat flows in a thermal gradient. Heat is randomized kinetic energy. As heat flows, thermal energy tends to disorganize forms. Forms tend to fly apart due to the randomized motions of their constituents. The only thing that holds the form together as a coherently organized self-replicating form is the potential energy of attractive forces. The life-form must feed upon a source of potential energy to maintain the organization of its form.

This actually gives a good definition of life-forms. A life-form is a self-replicating coherently organized form of information that must feed upon a source of energy in order to maintain the state of its organization in a recognizable form over a sequence of perceivable events. By this definition, a hurricane is a life-form. Not only must a life-form eat other forms in order to self-replicate form and survive as a recognizable form, but the life-form must also avoid being eaten by other forms. Life-form survival is really only a recognizable self-replication of form. What is called death is only an unrecognizable disorganization of form.

Why do self-replicating life-forms evolve in the world in the first place? The answer is inherent in the second law of thermodynamics, which says entropy tends to increase as heat flows in a thermal gradient. Life-forms have very low entropy, which means their forms must self-replicate within a small number of information configuration states for that self-replication of form to be recognizable, but they can only evolve in an environment as the total entropy of the life-form and its environment increases as heat flows. The total number of information configuration states for the life-form and its environment is actually increasing. The flow of heat and the increase in entropy are intrinsically related. Although not often appreciated, the life-form's environment is the observable universe. The big bang event that apparently created the observable universe was a very low entropy but a very high temperature state. As the universe expands in size from the big bang event, the universe cools in temperature but also increases in entropy.

This increase in entropy as the universe expands can be understood in terms of the holographic principle in terms of a cosmic horizon that defines the observable universe from the perspective of an observer at the central point of view, which is the singularity of the big bang event. The observer's cosmic horizon encodes all the bits of information for everything the observer can observe in its world. As the observable universe expands in size, the cosmic horizon increases in surface area, which means it encodes more bits of information, but also decreases in temperature, which creates the temperature gradient within which heat flows as entropy increases.

Life-forms are very efficient mechanisms for transferring heat in a thermal gradient, and as such, they are also very good mechanisms for increasing the entropy of their environment even as their own entropy remains low. A good example of this effect is photosynthesis in a plant. A plant consumes high energy low entropy visible photons that arrive from the sun and converts some of this energy into high energy low entropy molecules like carbohydrates, but in the process also radiates away many more lower energy higher entropy infrared photons into the environment. The thermal gradient within which photosynthesis takes place only arises because the sun is hot and the surface of the earth is cool, but outer space is even colder. The ultimate source of this thermal gradient is the expansion of the observable universe from the big bang event.

This dispersive mechanism of radiating away heat into the environment is a very efficient way of increasing the total entropy of the combined system of the life-form and its environment as heat flows in a thermal gradient. A life-form may be the most efficient dispersive mechanism possible for increasing total entropy, which may be why low entropy self-replicating life-forms naturally evolve as heat flows in a thermal gradient. This may be nature's way of maximizing the flow of heat and increasing entropy. A life-form that eats another life-form does exactly the same thing.

Like the complex formation of eddies and whirlpools in the flow of a river, the formation of self-replicating life-forms that eat each other in a struggle for survival may be the most efficient way nature has to maximize the flow of heat and increase entropy as heat flows in a thermal gradient. As long as heat flows through the world, life-forms spontaneously develop in the world just like whirlpools develop in the flow of a river. Even the geometric form of a spiral galaxy can be understood to develop in the flow of energy like the formation of a whirlpool.

Spiral Galaxies versus Whirlpools

This natural development of life-forms is an inevitable consequence of the observable universe expanding in size and cooling, which always appears to occur from the central perspective of an observer as that holographic world is defined on a cosmic horizon. The development and evolution of life-forms that eat each other in a struggle for survival may be an inevitable consequence of living in a holographic world, but only the life-forms really struggle for survival.

The observer only perceives this struggle for survival through the projection of life-form images from its holographic screen to its central point of view. The holographic principle is telling us that the fundamental nature of reality is not an observable world but the observer of that world. That world only consists of forms of information projected from a holographic screen, like movie images projected from a movie screen. The consciousness of the observer is more fundamental than the projected images. The consciousness of the observer is even more fundamental than the laws of physics that apparently govern events in that observable world.

The holographic principle as understood with non-commutative geometry in the context of dark energy tells us that ultimately the laws of physics are not encoded in reality. Instead, the laws of physics spontaneously emerge in the world as thermal averages as heat flows in a thermal gradient. In a similar way that water freezes into ice as heat flows away from water molecules, the laws of physics that apparently govern events in the observable universe spontaneously emerge or freeze out from the underlying state of potentiality that describes all possible laws of physics. This is an example of spontaneous symmetry breaking, which naturally occurs as dark energy burns away and the observable universe inflates in size and cools in temperature.

There are a lot of details about the standard model of particle physics that we do not understand, but these are only details. The overarching structure of both relativity theory as formulated with Einstein's field equations for the space-time metric and quantum field theories that represent the standard model of particle physics are quite simple and thermodynamically emerge in a natural way from the holographic principle and other geometric mechanisms. As Einstein remarked with his comments about his desire to know *the thoughts of God*, it is the overarching geometric structure that is important, not the details. There is nothing mysterious about how the laws of physics emerge in the world once we accept that the world is a holographic structure that arises through geometric mechanisms, like non-commutative geometry.

There is however a little problem with this scenario. If the laws of physics are not encoded in the ultimate nature of reality, what gives rise to the potential of all possible metastable false vacuum states that allows the physical universe to be created in the first place with a high value of dark energy and for the physical universe to make transitions to more stable lower energy vacuum states? What encodes for the metastable states? Physicists like to assume there is some grand theory of everything that encodes for these metastable vacuum states in terms of the cosmic landscape of that theory, but that kind of an assumption is not consistent with the holographic principle understood in the context of dark energy, which tells us the observer's accelerated frame of reference is the primary factor in the creation of the observer's world. This is inherently an observer-centric point of view in which the observer's cosmic horizon is observer-dependent. The observer must come first before the laws of physics can spontaneously emerge in the observer's world. The consciousness of the observer is primary, not the world the observer observes. Consciousness itself is the primary nature of existence, not an observable world.

This last statement about consciousness being the primary nature of existence seems to be a bit over the top, so it's worth examining what it really means. The holographic principle is telling us that all the fundamental bits of information that define an observable world as observed by the observer of that world are defined on a bounding surface of space that acts as a holographic screen. The bits of information are the fundamental dynamical degrees of freedom for everything that can appear to happen in that bounded region of space. It's also possible to define dynamical degrees of freedom for things that appear to exist within that bounded region of space, but these degrees of freedom, just like the things they represent, are not really fundamental. The degrees of freedom that appear to exist in a bounded region of space only result from holographic projection of forms of information from the observer's holographic screen to its central point of view.

The holographic projection of forms of information from the observer's holographic screen to its central point of view is like the projection of movie images from a movie screen to the point of view of an observer out in the audience. All the fundamental bits of information that characterize the movie are encoded on the screen. There really are no bits of information out in the audience. If the observer cannot be characterized in terms of bits of information, then what is the observer?

The observer can only be characterized as the perceiving consciousness that arises at a point of view. That central point of view always arises in relation to a surrounding holographic screen that encodes all the fundamental bits of information for whatever the observer can observe in its world. The observer's holographic screen is observer-dependent in the sense that it can only arise as an observation limiting event horizon within the observer's accelerated frame of reference. This is inherently an observer-centric view of the world, as the observer is at the central point of view of its own holographic world that is defined on a surrounding holographic screen. In reality, there are no bits of information at the observer's central point of view. Whatever dynamical degrees of freedom appear to exist at that point of view only result from holographic projection. All the fundamental degrees of freedom of the observer's holographic world are really defined on the observer's holographic screen. This way of understanding the nature of the world is an inevitable consequence of taking the holographic principle to its logical conclusion.

The observer can only be understood as the perceiving consciousness that arises at a point of view in relation to its own holographic screen. This raises an even more fundamental question:

What is the true nature of consciousness? Unlike all of modern physics, which is conceptually formulated in terms of mathematical concepts, this question cannot be answered conceptually.

The true nature of consciousness is not a form of information. That's what it means to say there really are no bits of information at the observer's central point of view. Every concept is a form of information. All the fundamental bits of information for the observer's world are encoded on its holographic screen. Since concepts are nothing more than forms of information, the observer's holographic screen is where all the concepts it has about the nature of its world must develop.

The true nature of the observer's consciousness cannot be conceptualized, precisely because it is not a form of information encoded on the screen. All of its concepts arise as forms of information encoded on the screen, but in-and-of-itself, consciousness does not. Consciousness is not a concept. It is exactly the other-way-around. Consciousness is what is perceiving, understanding and knowing about all concepts, but in-and-of-itself, consciousness cannot be conceptualized.

The only way to answer this question about what consciousness really is, is to go beyond concepts. It is the consciousness itself that must go beyond its concepts to know what it truly is. The direct experience that consciousness has of itself when it knows what it truly is, is called enlightenment. By its very nature, this is an experience that cannot be conceptualized.

As previously discussed, there are only two things about the nature of observable reality that we know with certainty from the testimony of enlightened beings, like Nisargadatta Maharaj. The first is that a fundamental description of observable reality must be observer-centric, in the sense the observer is at the central point of view of whatever it observes in its own observable world. The second is the nature of observation can best be described by the concept of holographic projection, in the sense that all observable forms of information are projected like images from a screen to the observer's central point of view. The irony is that an enlightened being would never refer to observable reality as *reality*, but rather as an illusion, like a dream or a virtual reality. Only the observer itself has an underlying or ultimate reality, which we call consciousness.

Why can't scientists see things clearly? Like everyone else who isn't enlightened, the vision of a scientist is clouded by delusion. The consciousness of a scientist identifies itself with something that it perceives, which is the form of a person. The result of seeing things with this twisted view of things is delusion. Only an enlightened being that has broken free of the bonds of personal self-identification and delusion can see things clearly. An enlightened being sees things for what they really are, which are illusions. An enlightened being also sees itself for what it really is, which is nothing but consciousness. The ultimate nature of being is consciousness.

The ultimate understanding one gains when one becomes enlightened is not conceptual. When one becomes enlightened, one realizes for oneself what one really is. One goes beyond all concepts of what one is to the true nature of one's being. The essential problem of enlightenment is that what one really is, is not a perceivable thing. Consciousness is not a perceivable thing and cannot be conceptualized as a form of information. The reason the experience of enlightenment cannot be conceptualized is because when one becomes enlightened, one knows nothing.

If consciousness cannot be conceptualized, then is it possible to describe what an observer experiences when the observer becomes enlightened? The answer is sort of, but not really, since enlightenment is the direct experience of the true nature of what consciousness is, and that cannot be conceptualized. Instead, all we can really do is discuss enlightenment in the sense of negation, or what consciousness isn't. Consciousness isn't anything it can perceive in its world.

The world consciousness perceives consist of forms of information projected like images from the observer's holographic screen to its central point of view. Everything perceivable is only a projected form of information. The forms of information are animated in the flow of energy like the frames of a movie. Like the form of things, the flow of energy is also perceivable. Concepts are also forms of information, but the flow of energy must create the context within which meaning is given to concepts. Both the form of things and the flow of energy that animates things can only be perceived if the observer has a holographic screen that projects animated images. The observer's holographic screen can only arise as an event horizon in the observer's accelerated reference frame. The observer only perceives things if the observer is accelerating.

What happens when this acceleration comes to an end. What happens when the observer stops accelerating? An observer that no longer accelerates is in a freely falling frame of reference. In the sense of the holographic principle, everything the observer can possibly observe only arises if the observer is in an accelerated frame of reference, since that is how the observer's holographic screen arises. When that acceleration comes to an end, there is no holographic screen.

What happens to the observer when the observer's acceleration comes to an end and the observer no longer has a holographic screen? Since everything the observer can observe in its world is a form of information that is encoded and organized on its holographic screen and is projected to its central point of view, when the observer stops accelerating, everything the observer perceives must disappear from existence. Even the observer must disappear from existence when there is no holographic screen, since the observer only arises in relation to its holographic screen.

When the observer stops accelerating, the observer enters into an ultimate freely falling frame of reference, and everything in the observer's world disappears from existence. This experience is often described as falling into the void. Even the observer itself disappears from existence, but consciousness does not stop existing. Consciousness is the ultimate nature of existence, and can never stop existing. Consciousness is what remains when everything else disappears from existence. What remains can be called the ground of being or the underlying nature of existence.

What remains when everything else disappears from existence is consciousness. In the sense of negation, or in terms of what it isn't, consciousness can only be described as nothingness, since in-and-of-itself, it is nothing perceivable. Consciousness is not something it can perceive. Since this ultimate state of existence cannot be conceptualized, it can be referred to as nonconceptual nothingness. Enlightenment is the direct experience that consciousness has of its true nature when it experiences nothing else, which can only be described as nonconceptual nothingness. Consciousness knows what it truly is by knowing itself to be that nonconceptual nothingness.

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