Article

Why the Proposed Second Law of Quantum Complexity is Mistaken

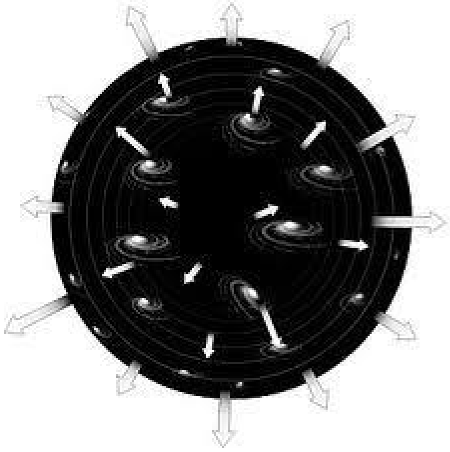
Abstract: Three arguments are given for why the proposed second law of quantum complexity is a mistaken idea: 1) the universe is not a closed system; 2) the universe is constructed in de Sitter space and not anti-de Sitter space; 3) observation of the universe disentangles the quantum state.

Keywords: Quantum complexity, quantum computing, thermodynamics, observation

Introduction: In a recent paper by Adam Brown and Leonard Suskind, a new second law of quantum complexity is proposed. This proposed second law states that quantum complexity will increase to its maximal value even when a system is classically at thermal equilibrium. Quantum complexity is a property of quantum computing, in which the qubits of information encoded in a quantum computer become increasingly entangled over the course of time even if the initial state of the qubits is disentangled. It typically takes an exponentially long time for all the qubits to become maximally entangled, which is characterized by a Kolmogorov exponent. The AdS/CFT correspondence is a holographic duality in which the gravitational properties of objects that form in anti-de Sitter space, like black holes, directly correspond to qubits of information encoded on the conformal boundary of anti-de Sitter space. In the AdS/CFT correspondence, increasing space-time volumes behind the event horizons of black holes that form in anti-de Sitter space are related to increasing quantum complexity due to increasing entanglement of the qubits encoded on the boundary of anti-de Sitter space. The basic idea is that even at thermal equilibrium, the behavior of black holes can change due to an increase in the complexity of the quantum state. In terms of the AdS/CFT correspondence, this increase in quantum complexity represents the increasing quantum entanglement of all the qubits encoded on the conformal boundary of anti-de Sitter space, which is reflected in the increasing internal space-time volumes of black holes that form in anti-de Sitter space. Thermal equilibrium is a classical concept that reflects the maximal thermal disorder or maximal entropy that occurs when all the dynamical degrees of freedom for some system carry the same amount of thermal energy, which is called the equipartition of energy. For a holographic world, those dynamical degrees of freedom are qubits encoded on a bounding surface of space. Isolated classical systems typically come into thermal equilibrium very quickly. While the classical thermal entropy of a black hole can increase to its maximal value in a very short period of time as thermal equilibrium is reached, quantum complexity only increases to its maximal value in an exponentially long time, which is the time required for the quantum state to become maximally entangled. Brown and Susskind propose that in addition to a second law of thermodynamics that describes a system achieving maximal classical thermal entropy, there also is a second law of quantum complexity that describes the quantum state of that system reaching a maximal state of quantum entanglement.

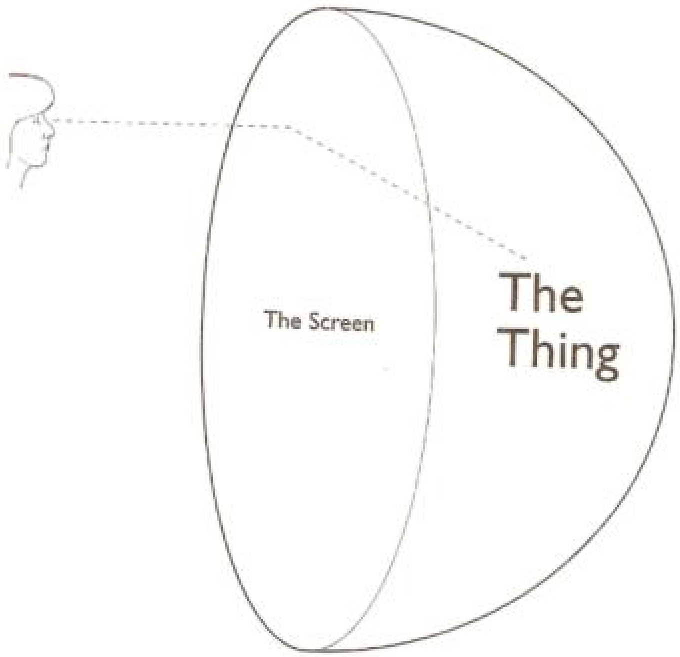
Analysis: I have three objections to the analysis of the Brown and Susskind paper:

1. The universe considered as a holographic world is not really a closed system. The AdS/CFT correspondence can be considered to be a closed system because of the nature of the conformal boundary of anti-de Sitter space, but the universe considered as a holographic world is not constructed in anti-de Sitter space, but rather in de Sitter space due to the effect of a positive cosmological constant. Every observer is at the central point of view of its own observed accelerated expansion of space, in which space appears to expand away from the observer's central point of view faster the farther the observer looks out into space, and so the observer's observations in space are always limited by its own de Sitter cosmic horizon, where space appears to expand away from the observer at the speed of light.



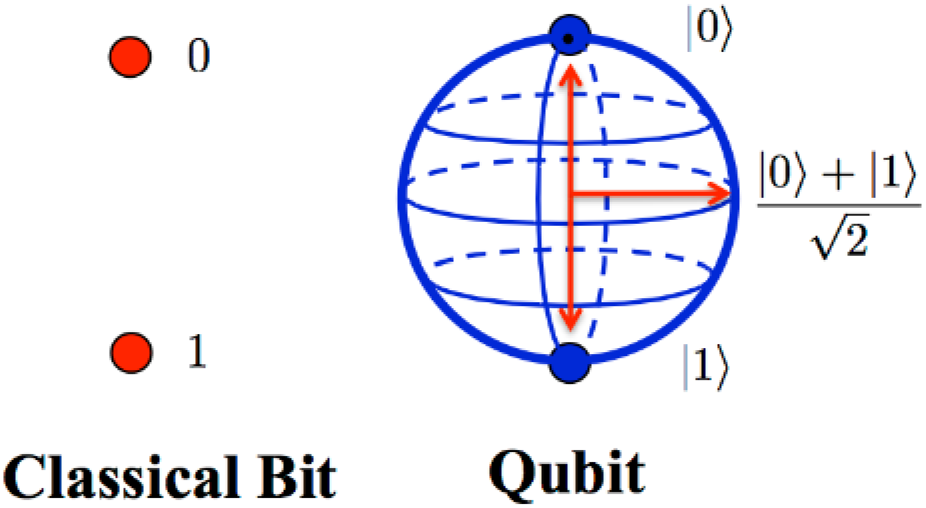
Accelerated Expansion of Space

If we encode qubits of information on the observer's de Sitter cosmic horizon along the lines of a matrix model, as formulated in the work of Tom Banks, all the observations of that holographic world are observer dependent. Everything observable in the observer's holographic world, which is an object of perception in the sense of a form of information, can be reduced to qubits of information encoded on the observer's own de Sitter cosmic horizon that acts as its holographic screen, just like the animated images of a movie that are projected from a computer screen to the point of view of an observer.

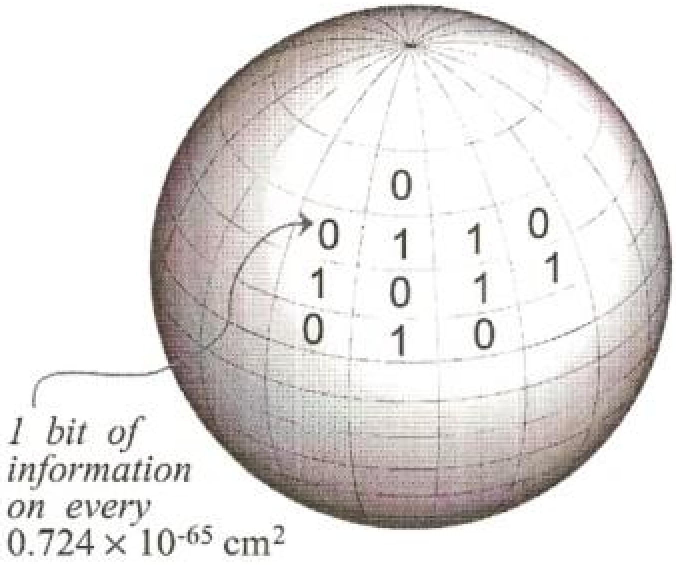


The Observer, the Screen, and its Object of Perception

A qubit is mathematically represented by a matrix, which is a two dimensional array of numbers. That two dimensional array of numbers must be encoded on a two dimensional surface of space, which is the observer's event horizon. The smallest possible event horizon is a Planck-size event horizon, which encodes a single qubit of information. Larger event horizons encode more qubits of information, but always in terms of an integral number of qubits. The total number of qubits encoded on the observer's horizon is proportional to the surface area of its horizon, just like the bits of information encoded on the pixels on a computer screen. The pixel size is the Planck area.

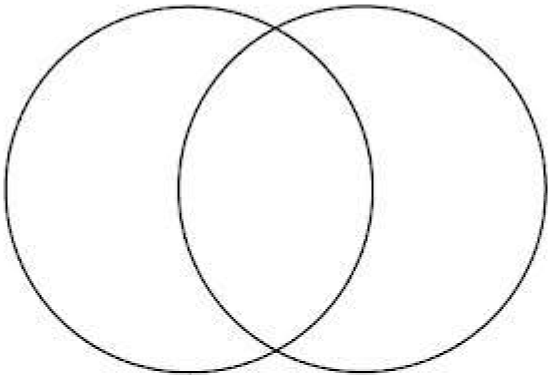


Qubit of Information Encoded on a Planck-size Event Horizon



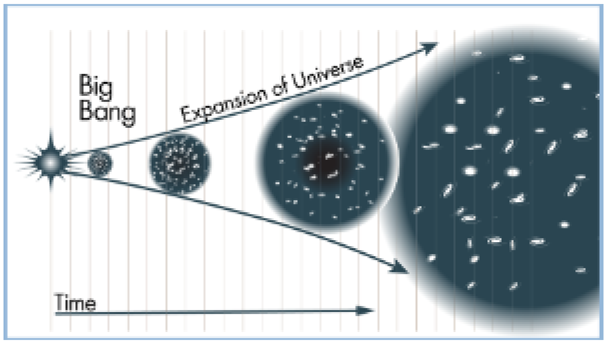
Holographic Principle

The observer's holographic world is defined on its own holographic screen that arises as its cosmic horizon due to the accelerated expansion of space, and its holographic screen projects all the animated images of that world to its central point of view, just like in a computer-generated virtual reality. Just like information sharing that occurs in a computer network of connected screens, multiple observers, each present at the central point of view of its own holographic world, can share in a consensual reality to the degree that their respective holographic screens overlap like a Venn diagram and share information.



Information Sharing Among Overlapping Holographic Screens

As Banks has argued, the cosmological constant is only a boundary condition for the construction of that holographic world. We have to assume a value for the cosmological constant, which sets the radius of the observer's cosmic horizon, before we can construct that holographic world. The inflationary cosmology theory of the big bang assumes that dark energy and the cosmological constant must transition from a higher to a lower value in order to explain the early history of the universe, which in effect resets the boundary condition. When the cosmological constant transitions to a lower value, the observer's cosmic horizon increases in radius and surface area, and by the holographic principle, encodes more qubits of information. Every such transition of the cosmological constant to a lower value in effect creates a new big bang event, which inherently is a state that is far away from thermal equilibrium and which tells us that the universe cannot be a closed system since the boundary condition changes.



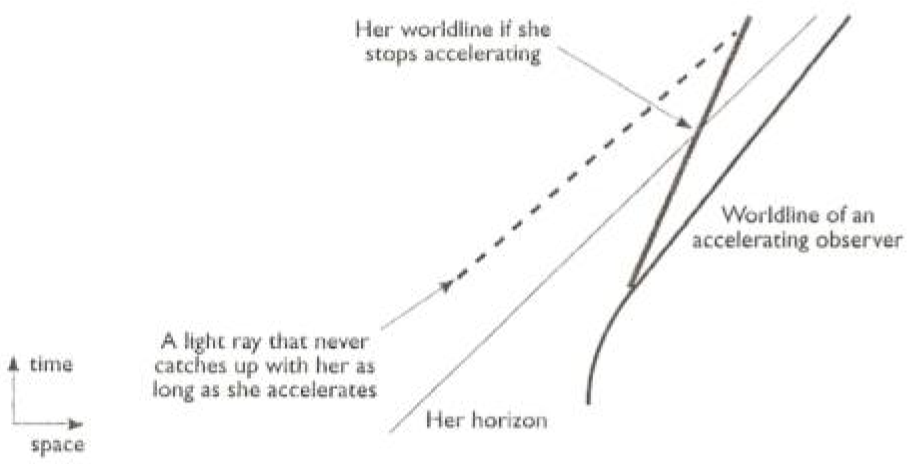
Accelerated Expansion of the Universe

2. The AdS/CFT correspondence relies on the idea of unitary time evolution, which is only a valid idea since the conformal boundary of anti-de Sitter space is a conformal Minkowski space in which the idea of time translation invariance is a valid concept. This means that all observers will agree upon the same definition of time.

$$|\Psi(t)\rangle = e^{-i\hat{H}t}\,|\Psi(0)\rangle$$

Unitary Time Evolution of the Quantum State

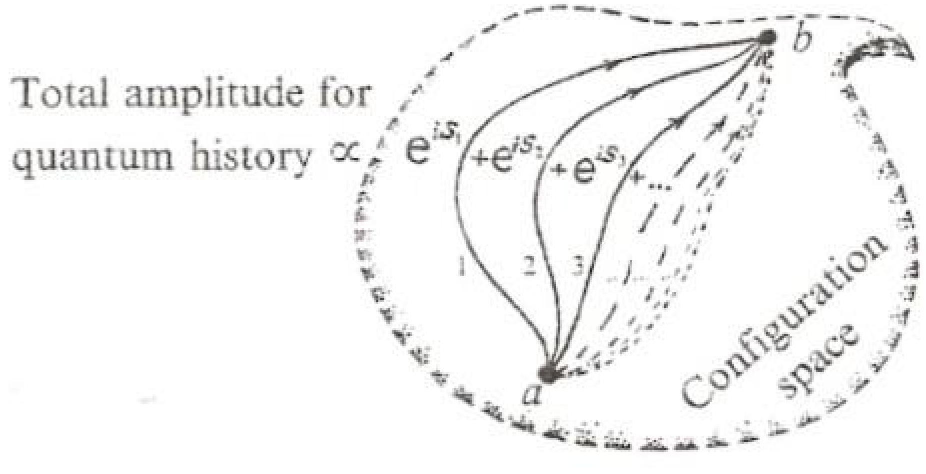
This is not the case in de Sitter space, where the only valid definition of time is the observer's own proper-time, which can vary from observer to observer based upon how different observers are moving relative to each other with accelerated motion. Every accelerating observer has its observation of events in space limited by its own event horizon, even when that acceleration takes place in de Sitter space due to the accelerated expansion of space. The observer's event horizon is always some combination of its de Sitter cosmic horizon and its Rindler event horizon that arises from its own accelerated motion. The observer's event horizon becomes its holographic screen when it encodes qubits of information.



Accelerating Observer's Event Horizon

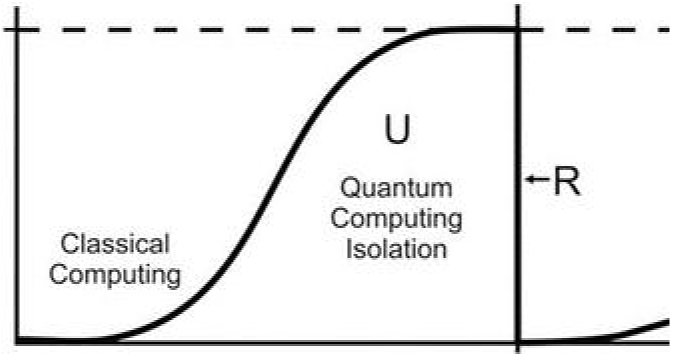
The whole idea of conventional quantum theory is based on the idea of unitary time evolution. which underlies the Feynman sum over all possible paths formulation of the quantum state. The problem is this is generally not a valid concept in any curved space-time geometry with gravity other than the AdS/CFT correspondence. In general, the idea of unitary time evolution is not a valid concept in a curved space-time geometry with gravity, like de Sitter space with gravity.

3. The above analysis ignores and does not take into account the effect of observation. In the AdS/CFT correspondence, for which unitary time evolution is a valid concept, each such observation can be considered to be an initial or final condition in the Feynman sum over all possible paths formulation of the quantum state.



Sum Over all Possible Paths Formulation of the Quantum State

Each initial or final condition is an observational event that disentangles the quantum state in the sense of a quantum state reduction, which is the nature of an observation. Roger Penrose has argued that observation must disentangle the quantum state through quantum state reduction.



Unitary Time Evolution versus Reduction of the Quantum State

Unitary time evolution tells us that the quantum state becomes increasingly entangled as it evolves between the initial and final states. In terms of qubits of information encoded on a bounding surface of space. like the conformal boundary of anti-de Sitter space or an observer's cosmic horizon in de Sitter space, the quantum state becomes increasingly entangled as it evolves in time and the qubits become increasingly entangled. The complexity of the quantum state only measures this degree of quantum entanglement of qubits, which become increasingly entangled over time from the initial disentangled state, which is a state of observation.

The initial state could be a state of thermal equilibrium, and the qubits will become increasingly entangled over time as the quantum state evolves from this initial disentangled state. Thermal equilibrium is best understood in terms of the equal partition of energy, which tells us that at thermal equilibrium, all the dynamical degrees of freedom for the system of interest carry the same amount of thermal energy given in terms of temperature as E=kT. For a holographic world, those dynamical degrees of freedom are qubits of information encoded on a bounding surface of space that arises as an observer's event horizon. If a total of n qubits encode information in a binary code, the maximal classical thermal entropy is given as S=kn, and there are a total of 2^n independent classical states, but at the quantum level, there are vastly more possible quantum states since the qubits can become entangled. Classical states are understood as eigenstates, while an entangled quantum state is a superposition of eigenstates. The classical states of qubits are the n eigenvalues of an n x n SU(2) matrix. The entanglement of qubits only represents rotational invariance on the surface of a 2-sphere. Thermal equilibrium only reflects that all the qubits carry the same amount of thermal energy given in terms of temperature. The initial state could be a state of thermal equilibrium, and yet the quantum state will evolve in time in terms of complexity from that initial disentangled state due to an increase in the degree of entanglement of all the qubits. The complexity of the quantum state only measures this degree of quantum entanglement of the qubits, which increases between observational events that disentangle the quantum state. That evolution of the quantum state continues until the next observational event, which is the final state in the sum over all possible paths that disentangles the quantum state.

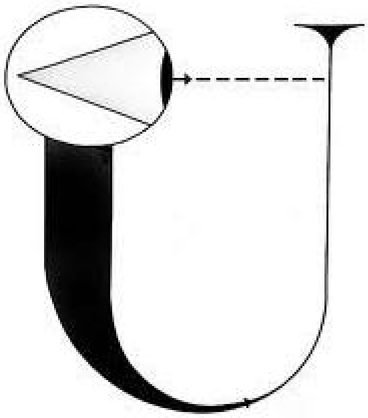
The essential problem with the proposed second law of quantum complexity is that it assumes that no observations are ever made. That is the only way the quantum state can continue to become increasingly entangled over the course of time. Observation disentangles the quantum state. The idea of thermal equilibrium and the maximal thermal entropy of a system is a classical concept, which by its very nature assumes that observations of the system are made and the quantum state is disentangled. The system must be observed in order to reach a maximal value of classical thermal entropy and come into thermal equilibrium. This directly contradicts the idea of quantum complexity increasing to a maximal value, which assumes that no observations are ever made that disentangle the quantum state. In the sum over all possible paths formulation of the quantum state, the initial and final states are disentangled states that correspond to observations.

In general, the initial and final states are not at thermal equilibrium, and the quantum state, as classically observed, evolves according to the laws of thermodynamics as heat flows from hotter to colder objects, which explains the normal flow of thermal energy through the observer's holographic world. This flow of heat is inherent in every observation the observer makes of its own holographic world. Only when the flow of heat comes to an end in a state of thermal equilibrium can we speak of the heat death of the universe, but that will never happen because the universe always has the option to recreate itself through another big bang event as the cosmological constant transitions to a lower value and the universe expands in size due to the accelerated expansion of space. The observable universe appears to expand in size due to the observer's comic horizon expanding in radius as the cosmological constant transitions to a lower value, which allows the whole thing to restart itself all over again as the observer's larger cosmic horizon cools in temperature and encodes more qubits of information.

Normal Flow of Thermal Energy Through the Observer's Holographic World

Associated with the flow of heat is the idea of thermal entropy increasing, which reflects the thermal disorganization of objects that occurs, as classically observed, due to the random flow of thermal energy. This thermal disorganization of objects is always counterbalanced by the tendency for the coherent organization of objects that arises because the entangled qubits that construct those objects, as coherently organized forms of information, tend to hold together over a sequence of observational events and self-replicate their forms in a recognizable way. The qubits have a natural tendency to bind together into forms due to quantum entanglement, like the entangled qubits that bind together in a spin network over a sequence of observational events.

The idea of the complexity of the quantum state only reflects the tendency the qubits have to become increasingly entangled as the quantum state evolves over time from some initial disentangled state. The idea of complexity increasing to a maximal value assumes that there is no observational event that disentangles the quantum state, which is not a valid assumption as long as there are observers around, like us, that make observations of the universe and thereby disentangle the quantum state through their observations of the universe. It just doesn't make any sense to speak of the quantum state as increasing in complexity to a maximal value when we're making observations of the universe. That's why a second law of quantum complexity makes no sense. It does make sense to speak of a second law of thermodynamics, which is only about the universe coming into thermal equilibrium in the sense of the equipartition of energy of all the qubits, but it makes no sense to speak of a second law of quantum complexity, where the quantum state becomes increasingly entangled until it reaches its maximum value of the entanglement of all the qubits, which assumes that no observations are ever made that disentangle the quantum state. It makes no sense since we are here as observers of the universe. With every observation, we disentangle the quantum state.



Universal Observer

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